Tutorial Assignments from Chapter 2

Note: Numbered problems are from the Chapter 2 Miscellaneous Problems section on page 96 of Edwards and Penney. Note also that for the problems numbered 34 or less, the directions are to either compute the limit or indicate that the limit doesn’t exist, and give an explanation.

6. \( \lim_{x \to 3} \frac{2x}{x^2 - x - 3} \)

10. \( \lim_{x \to 0} \frac{4x - x^3}{3x + x^2} \)

16. \( \lim_{x \to 1^+} (x - \sqrt{x^2 - 1}) \)

20. \( \lim_{x \to -2^-} \frac{x + 2}{|x + 2|} \)

26. \( \lim_{x \to 1^-} \frac{x}{x - 1} \)

30. \( \lim_{x \to 5^+} \frac{25 - x^2}{x^2 - 10x + 25} \)

34. \( \lim_{x \to 0} \frac{\tan 2x}{\tan 3x} \)

44. Find the slope of the tangent line to the function \( f(x) = 1 - 2x - 3x^2 \) at a generic point \( x = a \) by starting with the limit of the difference quotient. Then use this to find the equation of the tangent line at \( x = 1 \).

50. Find the slope of the tangent line to \( f(x) = \frac{1}{2x + 1} \) at the generic point \( x = a \).

56. Write an equation for a circle with center \((2, 3)\) which is tangent to the line with equation \( x + y + 3 = 0 \). Hint: call the point of tangency \((a, b)\). You know 2 things about this point. It is on the given line, and the line segment from \((a, b)\) to the center of the circle is perpendicular to the tangent line.

58. Where is \( f(x) = \frac{1-x}{(x^2-2x)^2} \) not continuous? Explain.

62. Use the intermediate value theorem to explain how you know that the equation \( x^3 - 4x^2 = 1 \) has three different solutions. Hint: Think about a function whose roots are the same as the solutions to this equation, and evaluate that function at some of your favorite small integers. What do your calculations (together with the intermediate value theorem) tell you about the roots of this function?