§7.1 Sets of Measure Zero

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Outline

Definition

Examples

Some theorems
**Definition**

A set $E \subseteq \mathbb{R}$ is said to be of **measure zero** provided that for each $\varepsilon > 0$ there exists a finite or a countable number of open intervals $I_1, I_2, I_3, \cdots$ such that

1. $E \subseteq \bigcup j I_j$, and
2. $\sum_j |I_j| < \varepsilon$.

**Theorem**

Let $x \in \mathbb{R}$. The set $E = \{x\}$ has measure zero.
**Theorem**

Let $x_1, x_2, \cdots, x_n \in \mathbb{R}$. The set $E = \{x_1, x_2, \ldots, x_n\}$ has measure zero.

**Theorem**

*The Cantor set has measure zero.*
Theorem

If $E \subset \mathbb{R}$ has measure zero and $A \subset E$, then $A$ has measure zero.

Theorem

If each of the sets $E_1, E_2, E_3, \cdots \subset \mathbb{R}$ is of measure zero, then $E = \bigcup_j E_j$ is also of measure zero.
Theorem

If $a, b \in \mathbb{R}$ with $a < b$, then none of the intervals $(a, b), [a, b), (a, b], or [a, b]$ has measure zero.