§15 Partitions

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Outline

1. What is a partition?

2. Counting classes

3. Additional problems
**Definition**

Let $A$ be a set. A **partition** of $A$ is a collection of nonempty, pairwise disjoint sets whose union is $A$.

**Example**

Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Let

$$P_1 = \{1, 4, 8, 10\}, \quad P_2 = \{2, 3\}, \quad P_3 = \{5\}, \quad P_4 = \{6, 7, 9\}$$

The collection $P = \{P_1, P_2, P_3, P_4\}$ forms a partition of $A$.

**Note**

- The collection of equivalence classes of a relation $R$ on a set $A$ form a partition of $A$.
- We will show that a partition of $A$ yields an equivalence relation on $A$. 
What is a partition?

**Definition**

Let $\mathcal{P}$ be a partition of the set $A$ and let $a, b \in A$. We say that $a$ is-in-the-same-partition-as $b$, denoted by

$$a \equiv_P b$$ provided that $\exists P \in \mathcal{P}, a, b \in P$.

**Theorem**

Let $\mathcal{P}$ be a partition of the set $A$.

1. $\equiv$ is an equivalence relation.
2. The equivalence classes of $\equiv$ are the parts of $\mathcal{P}$. (*Exercise 15.5*)

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**Counting classes**

**Counting cows**

If you want to count the number of cows in a field, count the number of legs and divide by 4.

**Figure:** A cow in a field
Theorem

Let $R$ be an equivalence relation on a finite set $A$. If all of the equivalence classes of $R$ have the same size, $m$, then the number of equivalence classes of $R$ is $|A|/m$.

Problem

How many distinguishable arrangements of the word ABBA are there?

Problem

How many distinguishable arrangements of the word BOOKKEEPER are there?
Problem
How many distinguishable arrangements of the word TAZEHIM are there if we require that the three vowels remain in order A before E before I.

Problem
How many distinguishable arrangements of the word HAZEHIM are there if we require that the three vowels remain in order A before E before I.

Problem
10 men are to be divided into two teams of 5 to play basketball. In how many ways can this be done?