§7 Counting Two-Element Lists

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Outline

1. Lists
2. The multiplication principle
3. The multiplication principle extended
4. Some helpful notation
5. A sample write-up
Lists

Definition

A list is an ordered sequence of objects. We will denote a list by separating the objects with commas and enclosing the objects in parentheses.

Problem

- What is the length of a list?
- When are two lists equal?
- What is an empty list?
The multiplication principle

Consider two element lists for which there are \( n \) choices for the first element, and for each choice of the first element there are \( m \) choices for the second element. Then the number of such lists is \( mn \).

Problem

*Five horses, \( \{A, B, C, D, E\} \), enter a race. In how many ways can they come in first and second place?*

Problem

*A keypad contains four keys, labeled \( \{a, b, c, d\} \). An access code consists selecting two keys from the pad in succession (repetition allowed). How many access codes are there?*

Problem

- *The 15 members of the Furman Fishing Club need to elect a president and a vice-president. In how many ways can this election take place?*
- *An experiment consists of tossing a coin and then a fair die, recording the outcomes in turn. How many outcomes does this experiment have?*
- *An experiment consists of tossing a red die and a blue die, recording the outcomes in turn. How many outcomes does this experiment have? How many of these outcomes do not contain a 6 on the blue die? How many of these outcomes do not contain a 3 or 4 on either the red or the blue die?*
The multiplication principle extended

Consider a list with \( k \) elements in which
- the first component has \( n_1 \) choices and
- for each \( j, \ 2 \leq j \leq k \), and independent of all of the previous choices, the \( j \)th component has \( n_j \) choices.

Then the number of such lists is

\[
n_1 \times n_2 \times n_3 \times \cdots \times n_k.
\]

Problem

- An experiment consists of tossing two coins in succession followed by tossing 3 dice in succession. In how many ways can this experiment be conducted?
- A baseball team consists of 17 players. In how many ways can the manager create a batting order?
- How many divisors (including 1 and itself) does 72 have?
- Acme Pizza Company makes custom order pizzas. There are three choices of crust and 5 toppings. To order a pizza you must select a crust and any number of the toppings, including none. How many different pizzas can be constructed?
Selecting from a pool

Suppose that a list of length $k$ is to be created by sampling (selecting) from a pool of size $n$.

- If the sampling is done with replacement (repetitions allowed), then there are $n \times n \times \cdots \times n = n^k$ elements in the list.

- If the sampling is done without replacement (no repetitions allowed), then there are $n \times (n-1) \times (n-2) \times \cdots \times (n-k+1) = (n)_k$ elements in the list. This last symbol is called a **falling factorial**.

Problem

- A MasterLock combination lock has 40 numbers on its dial, 0 through 39. A lock combination consists of a list of three of these numbers. How many lock combinations are there?

- A baseball team consists of 17 players. In how many ways can the manager create a batting order?
Problem
A baseball team consists of 17 players. In how many ways can the manager create a batting order?

Solution
A batting order is a list containing 9 players chosen from the set of 17. Since the list is to be constructed by sampling without replacement, it follows from the multiplication principle that the number of batting orders is

\[17 \times 16 \times 15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 = (17)_9.\]