1. In each of the following problems, you will be given a mathematical definition. Express this definition using the quantifiers $\forall$ and $\exists$, negate the definition, and write the negation of the definition in standard English.

(a) $f$ is integrable on $[a, b]$ provided that for every $\varepsilon > 0$ there is a partition $P$ of $[a, b]$ such that $U(P, f) - L(P, f) < \varepsilon$.

(b) The sequence $\{a_n\}$ converges to the $L$ provided that for every $\varepsilon > 0$ there is a natural number $N$ such that if $n \geq N$, then $|a_n - L| < \varepsilon$. 
2. Prove the following theorem: \( \forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, x + y = 0. \)

3. Prove or disprove: \( A \cup B = A \cap B \) if and only if \( A = B. \)

4. How many integers between 1 and 1000 inclusive are divisible by 5 or 13?
5. Let $O$ be the set of odd integers and let $A = \{x \in \mathbb{Z} : \exists k \in \mathbb{Z}, x = 4k + 3\}$. Show that $A \subset O$.

6. Prove the following Law of DeMorgan: Let $A$, $B$, and $C$ be sets. Then

$$A - (B \cup C) = (A - B) \cap (A - C).$$