§9.6–Hypothesis Tests for One Population Mean When σ is Unknown

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Outline

The test statistic

Suppose that a random sample of size n is drawn from a normal population with mean μ; let $\overline{x}$ and s be the sample mean and standard deviation. Then the distribution of

$$t = \frac{\overline{x} - \mu}{s/\sqrt{n}}$$

is a t-distribution with $n - 1$ degrees of freedom.

Problem

Courtesy of a University of North Carolina statistics web site.

An insurance company is reviewing its current policy rates. When originally setting the rates they believed that the average claim amount was $1,800. They are concerned that the true mean is actually higher than this, because they could potentially lose a lot of money. They randomly select 40 claims, and calculate a sample mean of $1,950 and a sample standard deviation of $496. At the 5% significance level, should the insurance company be concerned?

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- *Since our test is right-tailed, we will reject* H_0 *only if the test statistic,* t*, is large and positive. At* $\alpha = .05$ *and* $df = 39$*, the critical value is* $t_{.05} = 1.68$.

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- *Our non-rejection region is therefore* $(-\infty, 1.68)$ *and our rejection region is* $(1.68, \infty)$*.*
- *For the sample collected,* $t = 1.91$*. Since* t *falls into the reject region, we will choose to reject* H_0 *at this significance level.*

Problem

A family practice clinic claims that the amount of time a doctor spends with a patient is normally distributed with a mean of 14.5 minutes. A patient watchdog group, hearing complaints about patients being rushed through the clinic, randomly samples 25 patients and finds that sample mean of the visits was 14.14 minutes with a sample standard deviation of 1.4 minutes.

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- *At a significance level of* $\alpha = .05$*, can they charge the clinic with misrepresenting the lengths of their office visits?*
- *What is the p-value of the observed mean? (This requires* **R Commanders** *probability calculator.)*

Problem

Tests of older baseballs showed that when dropped 24 ft onto a concrete surface, they bounced an average of 235.8 cm. In a test of 40 new randomly selected baseballs, the bounce heights had a mean of 235.4 cm and a standard deviation of 4.8 cm. At the 5% significance, test the claim that the mean bounce heights of the new baseballs is different from 235.8 cm.

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- *What is the p-value of the observed mean?*

t-tests using R

It is easy to use **R Commander** to find confidence intervals and run hypothesis tests on a single sample of data. Once the data set has been loaded, select

$$\text{Statistics} \longrightarrow \text{Means} \longrightarrow \text{Single-sample t-test} \ldots$$

from the menus. You will be prompted to fill in the null hypothesis and the significance level (although in the menu, this is phrased in terms of the confidence, $1 - \alpha$). The results of these tests are printed to the screen.

Problem

ACME bottling company purchased a machine to fill bottles of water to 16 ounces. It strongly suspects that the machine is overfilling the bottles. A sample of 25 bottles was collected and their volumes were determined.

Table: ACME Sample Data

17.37	16.42	16.30	16.93	15.54
15.64	15.93	16.34	15.91	16.88
16.01	16.49	15.75	15.83	16.32
16.76	15.95	16.35	15.25	16.62
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- *What is the p-value of the observed data?*