§8.4–Confidence Intervals for One Population Mean When σ is Unknown

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Fall Term 2009

Outline

If σ is known

If we draw a random sample of size n from a population with mean μ and standard deviation σ, then the distribution of the sample mean, $\overline{x}$, is normal with mean μ and standard deviation $\sigma/\sqrt{n}$. In other words,

$$z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}}$$

has a standard normal distribution.

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- If we draw a random sample of size n from a population with mean μ and unknown standard deviation, then the statistic

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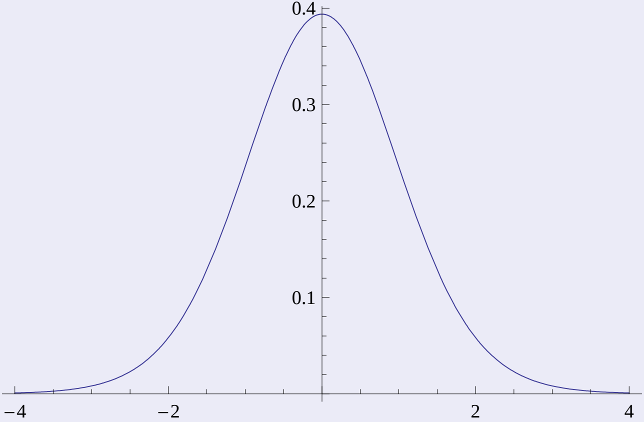
  called the studentized version of $\overline{x}$ is not normally distributed.
- The distribution of t is called a t-distribution with $n - 1$ degrees of freedom.

A *t*-distribution curve

Here is a picture of a *t*-distribution curve with 19 degrees of freedom.



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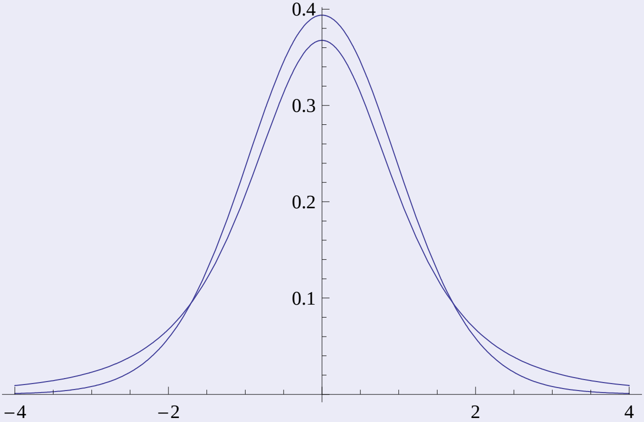
- A *t*-curve is always positive, but it tends toward 0 in both the positive and negative directions.
- A *t*-curve is symmetric about the *y*-axis.
- As the number of degrees of freedom becomes larger, *t*-curves look increasingly like the standard normal curve.

A comparison of *t*-distribution curves

Here are pictures of the graphs of two *t*-distribution curves. The squat curve has 3 degrees of freedom and the other has 19 degrees of freedom.



Area under the t-curve

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Distributions → Continuous distributions
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- *Find the area under the t-curve with 26 degrees of freedom to the right of* 2.25.
- *Find the area under the t-curve with 11 degrees of freedom between* -1.35 *and* 1.86.

The quantity t_α

- Let $0 < \alpha < 1$. Given a number of degrees of freedom, t_α denotes the number such that the area under the corresponding t-curve to the right of t_α is α.
- This can be done by **R Commander** using

  Distributions $\rightarrow$ Continuous distributions

  $\rightarrow$ t distribution $\rightarrow$ t quantiles ...

  Upon filling in the dialog box, the desired value of t_α will be issued.

Problem

Find the value of $t_{.05}$ for a t-distribution with 5 degrees of freedom. Use **R Commander** *and the table in the text.*

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- In other words, the chance that the actual mean falls outside of this interval is α; this represents our risk in using the stated confidence interval to estimate μ.

Newcomb's "speed of light" data

In 1878, Simon Newcomb worked out an ingenious method for measuring the speed of light. He measured the time (in nanoseconds) for light to travel from the United States Naval Observatory to the Washington Monument and back, a distance of some 7400 meters. His measurements are tabulated below. These measurements represent deviations from 24,800 nanoseconds.

28	26	33	24	34	-44	27	16	40	-2
29	22	24	21	25	30	23	29	31	19
24	20	36	32	36	28	25	21	28	29
37	25	28	26	30	32	36	26	30	22
36	23	27	27	28	27	31	27	26	33
26	32	32	24	39	28	24	25	32	25
29	27	28	29	16	23				

Find a 90% confidence interval for the mean of this data.