Assignment #5 **Name** Answer Key

Due 18 April 2008

1. Find $p_1(x, y, z)$ and $p_2(x, y, z)$ for the function $f(x, y, z) = xe^{2y-z}$ centered at $a = (0, 0, 0)$. Express your answers as polynomials; thus, evaluate the dot products and matrix multiplications. Collect like terms. Evaluate $p_1(.1, .2, .1)$ and $p_2(.1, .2, .1)$. Which polynomial gives the better approximate value to $f(.1, .2, .1)$?

$f(0,0,0) = 0$

$\frac{\partial f}{\partial x} = e^{2y-z} \qquad \frac{\partial f}{\partial y} = xe^{2y-z}(2) \qquad \frac{\partial f}{\partial z} = xe^{2y-z}(-1)$

$\nabla f(0,0,0) = (1, 0, 0).$

$p_1(x, y, z) = 0 + (1, 0, 0) \cdot (x, y, z)$

$p_1(x, y, z) = x.$

$$Hf(x, y, z) = \begin{bmatrix} 0 & 2e^{2y-z} & -e^{2y-z} \\ 2e^{2y-z} & 4xe^{2y-z} & -2xe^{2y-z} \\ -1e^{2y-z} & -2xe^{2y-z} & xe^{2y-z} \end{bmatrix}$$

$$Hf(0,0,0) = \begin{bmatrix} 0 & 2 & -1 \\ 2 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$p_2(x, y, z) = x + \frac{1}{2}(x, y, z)\begin{bmatrix} 0 & 2 & -1 \\ 2 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$p_2(x, y, z) = x + 2xy - xz$

$p_1(.1, .2, .1) = .1$; $p_2(.1, .2, .1) = .13$; $f(.1, .2, .1) \cong .13498$

p_2 is the better approximation.

2. Let a, b, and c be real numbers such that $a \neq 0$ and $b^2 - 4ac > 0$, and let $f(a,b,c)$ denote the *distance* between the roots of the corresponding quadratic equation $ax^2 + bx + c = 0$.

(a) Find a formula for $f(a,b,c)$ and evaluate $f(1,2,-3)$.

$$f(a,b,c) = \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right) - \left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$$

$$= \frac{\sqrt{b^2 - 4ac}}{a}$$

$$f(1,2,-3) = \frac{\sqrt{4 - 4(1)(-3)}}{1} = \frac{\sqrt{16}}{1} = 4.$$

(b) Find an expression for the total differential df at the point $(1,2,-3)$ for $h = (da, db, dc)$. To which entry, a, b, or c, is the value of f most sensitive?

$$\frac{\partial f}{\partial a} = \frac{a\,\frac{1}{\cancel{2}}(b^2 - 4ac)^{-\frac{1}{2}}(-\cancel{4}^{2}c) - \sqrt{b^2 - 4ac}\,(1)}{a^2}$$

$$\frac{\partial f}{\partial a}(1,2,-3) = \frac{+6}{4} - 4 = -5/2$$

$$\frac{\partial f}{\partial b} = \frac{1}{a} \cdot \frac{1}{\cancel{2}}(b^2 - 4ac)^{-\frac{1}{2}}(\cancel{2}b)$$

$$\frac{\partial f}{\partial b}(1,2,-3) = \frac{2}{(1)(4)} = \frac{1}{2}.$$

$$\frac{\partial f}{\partial c} = \frac{1}{\cancel{a}} \frac{1}{\cancel{2}}(b^2 - 4ac)^{-\frac{1}{2}}(-\cancel{4}^{2}\cancel{a})$$

$$\frac{\partial f}{\partial c}(1,2,-3) = \frac{-2}{4} = -\frac{1}{2}.$$

$$df\big((1,2,3), (da, db, dc)\big) = -\frac{5}{2}\,da + \frac{1}{2}\,db - \frac{1}{2}\,dc.$$

f is most sensitive to a, since $\frac{\partial f}{\partial a}$ dominates