Assignment #2 **Name** Answer Key

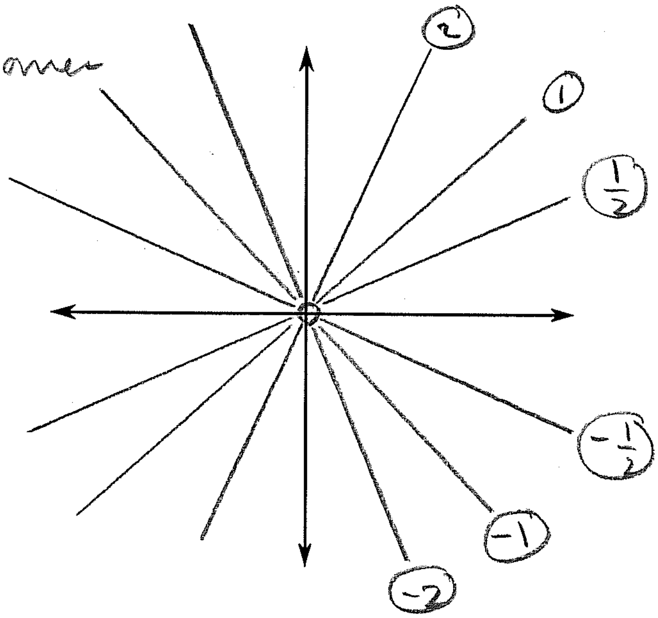
Due 14 March 2008

1. Let $f(x,y) = y/x$.
   (a) What is the domain of f? $D = \{(x,y) : x \neq 0\}$.

   (b) Sketch the level curves of f corresponding to the levels $c = 0, \pm 1/2, \pm 1, \pm 2$; label the level curves accordingly.

   $\frac{y}{x} = c$ becomes

   $y = cx$.



   (c) What is the range of f? range $= (-\infty, \infty)$ or $\mathbb{R}$.

2. In each case, evaluate the limit or show that it does not exist.

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^4+y^4}$

   $\displaystyle\lim_{(x,0)\to(0,0)} \frac{x(0)^3}{x^4+0^4} = 0$ But

   $\displaystyle\lim_{(x,x)\to(0,0)} \frac{x^4}{x^4+x^4} = \frac{1}{2}$ $\therefore$ the limit does not exist.

   (b) $\displaystyle\lim_{\substack{(x,y)\to(1,1)\\ x\neq y}} \frac{x^3-y^3}{x^2-y^2} = \lim_{\substack{(x,y)\to(1,1)\\ x\neq y}} \frac{\cancel{(x-y)}(x^2+xy+y^2)}{\cancel{(x-y)}(x+y)}$

   $$= \frac{1^2 + (1)(1) + 1^2}{1+1} = \frac{3}{2}.$$

3. Let $f(x,y) = \dfrac{x^2y^2}{x^2+y^2}$ for $(x,y) \neq (0,0)$. Can we define $f(0,0)$ so as to make f continuous at $(0,0)$? Explain.

We can define $f(0,0)$ to make f continuous @ $(0,0)$ provided the limit exists. We will use polar coordinates.

$$\lim_{r\to 0} \frac{r^2\cos^2\theta \, r^2 \sin^2\theta}{r^2} = \lim_{r\to 0} r^2 \cos^2\theta \sin^2\theta = 0$$

Thus we should set $f(0,0) = 0$ to make f continuous @ $(0,0)$.

4. Find the equation of the plane tangent to the surface $f(x,y) = \sqrt{169 - x^2 - y^2}$ at the point on the surface corresponding to $(x,y) = (3,4)$.

$$f(3,4) = \sqrt{169 - 3^2 - 4^2} = \sqrt{144} = 12.$$

$$\nabla f(x,y) = \left(\tfrac{1}{2}(169 - x^2 - y^2)^{-\frac{1}{2}}(-2x), \ \tfrac{1}{2}(169 - x^2 - y^2)^{-\frac{1}{2}}(-2y) \right)$$

$$= \left(\frac{-x}{\sqrt{169 - x^2 - y^2}}, \frac{-y}{\sqrt{169 - x^2 - y^2}} \right)$$

$$\nabla f(3,4) = \left(\frac{-3}{12}, \frac{-4}{12} \right) = \left(-\frac{1}{4}, -\frac{1}{3} \right).$$

The equation of the tangent plane is

$$z = h(x,y) = 12 + \left(-\tfrac{1}{4}, -\tfrac{1}{3}\right) \cdot (x-3, y-4)$$

$$= 12 + \left(-\tfrac{1}{4}\right)(x-3) + \left(-\tfrac{1}{3}\right)(y-4).$$