### 3.7

#### 2(a)

#### (b)

#### (c) \[ V = (3-2x)(3-2x) x = x(3-2x)^2, \quad 0 < x < \frac{3}{2} \]

#### (d)-(e) done already.

#### (f) \[
V' = \left( 3-2x \right)^2 + x(2)(3-2x)(-2) \\
= (3-2x) \left( 3-2x - 4x \right) \\
= (3-2x)(3-6x)
\]

\[ x = \frac{3}{2}, \quad x = \frac{1}{2} \]

Outside of domain: \( x = \frac{1}{2} \)

Check: \[ V''(x) = -2(3-6x) + (3-2x)(-6) \]

\[ V''(1/2) = -2(3-3) + (3-1)(-6) < 0 \]

\( \circ \) x = 1/2 produces a max. volume

**Answer:** the largest volume that such a box can have is\[ V(1/2) = 2 \text{ cubic feet} \]
#30. cylinder inside a cone.

Cross-section view:

\[ x = \text{radius of cylinder} \]
\[ y = \text{height of cylinder} \]
\[ V = \pi x^2 y \text{ objective} \]

similar triangles,

\[ \frac{y}{r-x} = \frac{h}{r} \]
\[ y = \frac{h}{r}(r-x) \text{ constraint} \]

\[ V = \pi x^2 \frac{h}{r} (r-x) = \left( \frac{\pi h}{r} \right) x^2 (r-x) \]
\[ 0 < x < r \]

Note: these are all constant for this problem.

\[ V' = \left( \frac{\pi h}{r} \right) \left[ 2x(r-x) + x^2 (-1) \right] = \left( \frac{\pi h}{r} \right) x(2r-3x) \]
\[ v' = 0 \Rightarrow x \neq 0 \text{ or } x = \frac{2}{3}r \]

not in the domain

Check: \[ v''(x) = \left(\frac{\pi h}{r}\right) \left[ 2r - 3x + x(-3) \right] \]

\[ v''\left( \frac{2r}{3} \right) = \left(\frac{\pi h}{r}\right) \left[ 2r - 3\left(\frac{2r}{3}\right) - 3\left(\frac{2r}{3}\right) \right] \]

\[ = \frac{\pi h}{r} (-2r) < 0 \]

Thus \( \frac{2r}{3} \) locates a max. value for the volume

Answer: The largest possible volume is

\[ V\left( \frac{2r}{3} \right) = \left(\frac{\pi h}{r}\right) \left( \frac{2r}{3} \right)^2 \left( r - \frac{2}{3}r \right) \]

\[ = \frac{\pi h}{r} \left( \frac{4r^2}{9} \right) \left( \frac{1}{3} + \frac{1}{3} \right) \]

\[ = \frac{4\pi}{27}r^2h \]

5. 39

12. \( g(x) = 5 - 4x^3 + 2x^6 = 5x^{-6} - 4x^3 + 2 \)

\[ G(x) = 5x^{-5} - 4x^{-2} + 2x + C \]

\[ = -x^{-5} + 2x^{-2} + 2x + C \]
16. \( f(x) = 6x^2 - 7 \tan^2(x) \)

\[
F(x) = \frac{6x^3}{3} - 7 \tan(x) + C
\]

\[
F(x) = 2x^3 - 7 \tan(x) + C
\]

28. \( f'(x) = 5x^4 - 3x^2 + 4, \quad f(-1) = 2 \)

\[
f(x) = \frac{5x^5}{5} - \frac{3x^3}{3} + 4x + C
\]

\[
f(x) = x^5 - x^3 + 4x + C
\]

\[
2 = f(-1) = -4 + C \quad \therefore \quad C = 6
\]

\[
f(x) = x^5 - x^3 + 4x + 6
\]

38. \( f''(x) = 20x^3 + 12x^2 + 4, \quad f(0) = 8 \)

\[
f'(x) = \frac{20x^4}{4} + \frac{12x^3}{3} + 4x + C
\]

\[
f'(x) = 5x^4 + 4x^3 + 4x + C
\]

\[
f(x) = \frac{5x^5}{5} + \frac{4x^4}{4} + \frac{4x^2}{2} + Cx + D
\]

\[
f(x) = x^5 + x^4 + 2x^2 + Cx + D
\]

\[
f(x) = 5 + 8 + 2 \cdot 1 + C \cdot 1 + D
\]

\[
8 = f(0) = D \quad \therefore \quad D = 8
\]
\( f(x) = x^5 + x^4 + 2x^2 + Cx + 8 \)

\[ 5 = f(1) = 12 + C \quad \therefore \quad C = -7 \]

\( f(x) = x^5 + x^4 + 2x^2 - 7x + 8 \)

#60. The two equations of position are

\('#1: y(t) = -16t^2 + 48t + 432
\]
\n\('#2: y(t) = -16(t-2)^2 + 24(t-2) + 432
\]

Note: that we need \( t-2 \) here because this ball is launched 2 seconds later.

Facts about ball #1

\[ y'(t) = -32t + 48 = 0 \]
\[ t = \frac{48}{32} = 1.5 \text{ seconds} \]

This ball #1 is beginning to fall before ball #2 is launched.

\[ y(t) = 0 \quad @ \quad t \approx 6.9 \text{ seconds} \]

On the other hand

\[ Y(t) = 0 \quad @ \quad t \approx 8 \text{ seconds} \]

So the first ball must pass the second ball on the way down.
#66.

\[a(t) = -22 \text{ ft/sec}^2\]

\[v(t) = -22t + C\]

Note: in order to solve for \(C\), we need to convert 50 mi into feet per second.

\[
\frac{50 \text{ mi}}{\text{hr}} = \frac{(50)(5280) \text{ ft}}{(3600) \text{ sec}} \approx 73.3 \text{ ft/sec}.
\]

\[v(t) = -22t + 73.3\]

\[x(t) = -11t^2 + 73.3t + C \quad x_0 = 0\]

\[x(t) = -11t^2 + 73.3t\]

The car comes to rest when \(v(t) = 0\) or \(-22t + 73.3 = 0\) or \(t = 73.3/22 \approx 3.3 \text{ seconds}\)

\[x(3.3) = -11(3.3)^2 + 73.3(3.3) \approx 127 \text{ ft}.\]