§5.1–Areas Between Curves

Tom Lewis

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Outline

An overview of the problem

Some examples of the method
An overview of the problem

Let $f$ and $g$ be integrable functions on an interval $[a, b]$ and suppose that $f \geq g$ throughout the interval. What is the area of the region trapped between these curves?

![Graph of functions $f(x)$ and $g(x)$]

Theorem

If $f$ and $g$ are integrable on $[a, b]$ and if $f(x) \geq g(x)$ for $a \leq x \leq b$, then the area of the region trapped between $f$ and $g$ over the interval $[a, b]$ is

$$\int_{a}^{b} (f(x) - g(x)) \, dx.$$
Proof.

- Let $P = \{x_0, x_1, \ldots, x_n\}$ be a partition of the interval $[a, b]$ and let $\{x_i^* : 1 \leq i \leq n\}$ be a set of sample points.
- On each subinterval $[x_{i-1}, x_i]$, the area trapped between $f$ and $g$ can be approximated by the area of a rectangle with height $f(x_i^*) - g(x_i^*)$ and width $\Delta x_i$. Thus the area of the region can be approximated by the Riemann sum:

\[
\text{Area} \approx \sum_{i=1}^{n} (f(x_i^*) - g(x_i^*)) \Delta x_i
\]

- Taking limits as $\|P\| \to 0$, we may conclude that

\[
\text{Area} = \int_{a}^{b} (f(x) - g(x)) \, dx.
\]
Problem

*Find the area of the bounded region between $y = x$ and $y = x^2$.***
Problem

Find the area of the bounded region between the curves $y^2 = x$ and $y = x - 2$.

Problem

Let $R$ be the region bounded by $y = x$, $y = 3 - 2x$, and the $x$-axis. Find the area of $R$ by . . .

1. integration with respect to $x$
2. integration with respect to $y$