§4.4–Indefinite Integrals and the Net Change Theorem

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Outline

Indefinite integrals

The Net Change Theorem

Applications
Definition
The symbol $\int f(x) \, dx$, called the **indefinite integral of $f$**, stands for the most general form of an antiderivative of $f$. Thus if $F$ is an antiderivative of $f$, then

$$\int f(x) \, dx = F(x) + C,$$

where $C$ is a constant.

Problem
*Evaluate the following integrals:*

1. $\int \frac{x^2 + x^6}{x^4} \, dx$
2. $\int (\sec(x) \tan(x) + \sin(x)) \, dx$
Theorem (The Net change Theorem)

The integral of the rate of change is the net change. In symbols,

\[ \int_{a}^{b} f'(t) \, dt = f(b) - f(a). \]

Problem

Water is being added to a tank at a rate of \( r(t) = 10 - t \) gallons per minute. How much water is added to the tank between \( t = 2 \) and \( t = 6 \) minutes?
Theorem

The area trapped between \( f(x) \) and the \( x \)-axis on the interval \([a, b]\) is \( \int_{a}^{b} |f(x)| \, dx \).

Problem

Find the area trapped between \( y = -x^2 + 3x - 2 \) and the \( x \) axis on the interval \([0, 4]\).
**Theorem**

Let \( v(t) \) be the velocity of a particle traveling in a line. Then

- \[ \int_a^b v(t) \, dt = s(b) - s(a), \] which is the displacement of the particle over the time interval \([a, b]\).

- \[ \int_a^b |v(t)| \, dt = \text{the total distance traveled by the particle on the time interval} \ [a, b]. \]

**Problem**

The velocity of particle is given by \( v(t) = \cos(t) \). Find the displacement and total distance traveled by the particle over the time interval \([0, 3\pi/2]\).