§4.3–The Fundamental Theorem of Calculus

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Outline

The signed-area function

The Fundamental Theorem of Calculus (FTC)

The Evaluation Theorem

Examples
Definition
The function
\[ g(x) = \int_a^x f(t) \, dt \]
gives the signed area between the curve \( y = f(x) \) and the \( x \)-axis on the interval \([a, x]\).

Problem
Work problem \#2 from §4.2 homework, p. 318.
Theorem (The Fundamental Theorem of Calculus)

If $f$ is continuous on the interval $[a, b]$, then

$$
\frac{d}{dx} \int_{a}^{x} f(t) \, dt = f(x) \quad \text{for all } a < x < b.
$$

Problem

Evaluate each:

- $\frac{d}{dx} \int_{0}^{x} \sqrt{1 + t^2} \, dt$
- $\frac{d}{dx} \int_{0}^{x} \frac{1}{1 + t^2} \, dt$
- $\frac{d}{dx} \int_{0}^{x} \tan^2(t) \csc(t) \, dt$
Theorem (Leibniz’ Theorem)

\[
\frac{d}{dx} \int_a^{u(x)} f(t) \, dt = f(u(x)) \cdot u'(x).
\]

Problem

Evaluate each of the following derivatives:

- \( \frac{d}{dx} \int_1^{x^2} \frac{1}{1 + t^2} \, dt \)
- \( \frac{d}{dx} \int_{\tan(x)}^{0} \sqrt{1 + t^2} \, dt \)
- \( \frac{d}{dx} \int_{2x}^{3x} \sin^2(t) \, dt \)
Theorem (The Evaluation Theorem)

If \( f \) is continuous on the interval \([a, b]\), then

\[
\int_{a}^{b} f(x) \, dx = F(b) - F(a),
\]

where \( F \) is any antiderivative of \( f \), that is, \( F' = f \).

Notation

If \( F \) is an antiderivative of \( f \), then we often write

\[
\int_{a}^{b} f(x) \, dx = F(x) \bigg|_{a}^{b} = F(b) - F(a)
\]

Problem

Solve the following integrals:

1. \( \int_{0}^{8} 3\sqrt{x} \, dx \)
2. \( \int_{1}^{r} \frac{y + 5y^7}{y^3} \, dy \)
3. \( \int_{\pi/6}^{\pi/3} \csc(t) \cot(t) \, dt \)
4. \( \int_{0}^{\pi} |\cos(x)| \, dx \)