Change problem #7 P(1,3). 11

Test #2 Name Answer Key

Directions: Show all of your work. Use proper notation.

1. Use the graph of $f(x) = 1/x$ to find a number $\delta > 0$ such that

$$\text{if } |x-2| < \delta, \text{ then } |f(x) - .5| < .3.$$

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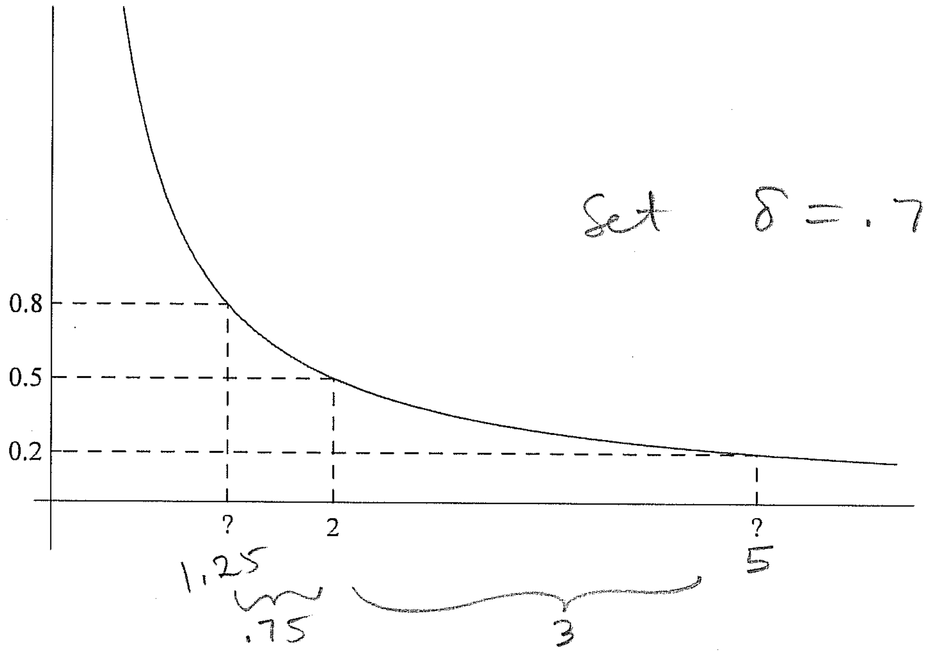
Set $\delta = .75$



2. Ted ran around a track and his time and distance were recorded in ten-second intervals. The data was collected in the table below; s denotes the distance traveled (in meters) and t denotes the elapsed time (in seconds):

t	0	10	20	30	40	50	60	70
s	0	65	135	195	270	330	395	445

(a) What is Ted's average velocity (meters/sec) for the entire 70 second run?

3 $\frac{445}{70} = 6.36 \ \frac{m}{s.}$

(b) Of the 7 ten-second time intervals, $[0, 10], [10, 20], \ldots, [60, 70]$, during which one was Ted's average velocity the greatest? During which one was Ted's average velocity the least?

3 Ted is slowest on $[60, 70]$: $\frac{50}{10} = 5 \ \frac{m}{s.}$

Ted is fastest on $[30, 40]$: $\frac{75}{10} = 7.5 \ \frac{m}{s.}$

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3. Sketch the graph of a function satisfying the following conditions:

- f is not defined at 0
- $f(-1) = 1$
- $f(1) = 2$
- $\lim_{x\to 1^+} f(x) = 2$
- $\lim_{x\to 1^-} f(x) = 1$
- $\lim_{x\to 0} f(x) = 0$
- $\lim_{x\to 2} f(x) = +\infty$

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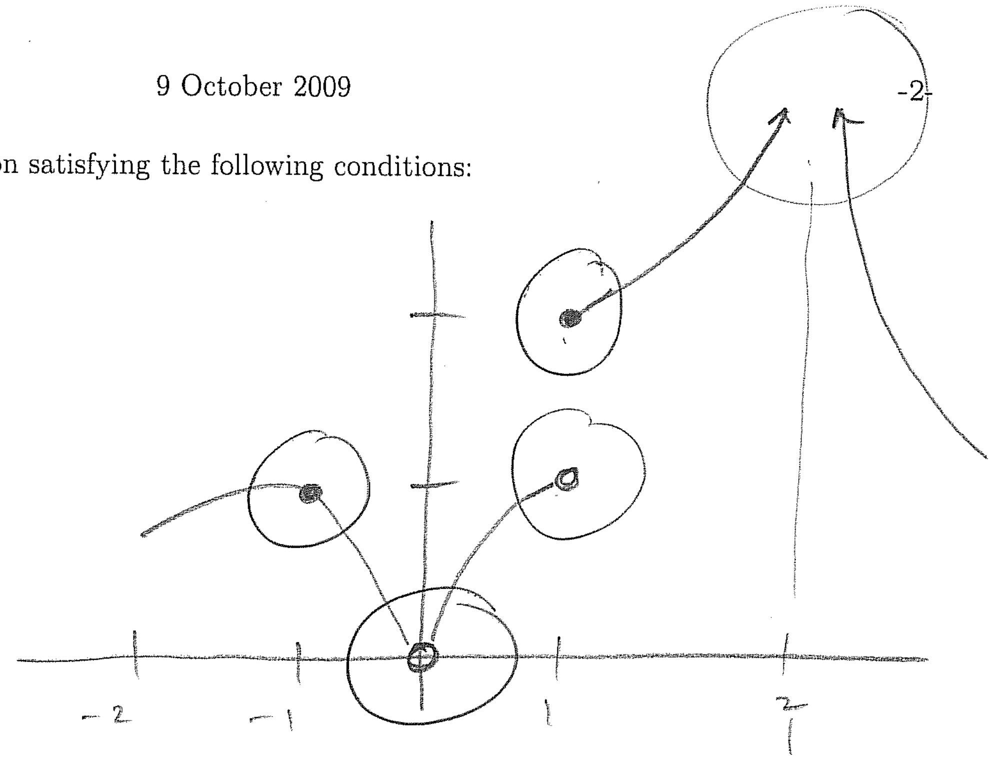
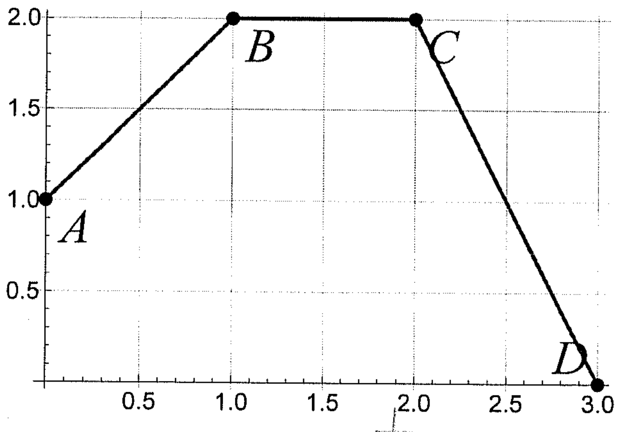


4. The graph of a function $y = f(x)$ is given below. Sketch the graph of the function $y = 2f(x+1)+3$. Locate the images of the points A, B, C, and D under this transformation.



① shif left by 1
② scale in y by 2
③ shift up by 3.

A (0,1) → (-1,1) → (-1,2) → (-1,5
B (1,2) → (0,2) → (0,4) → (0,7)
C (2,2) → (1,2) → (1,4) → (1,7)
D (3,0) → (2,0) → (2,0) → (2,3)

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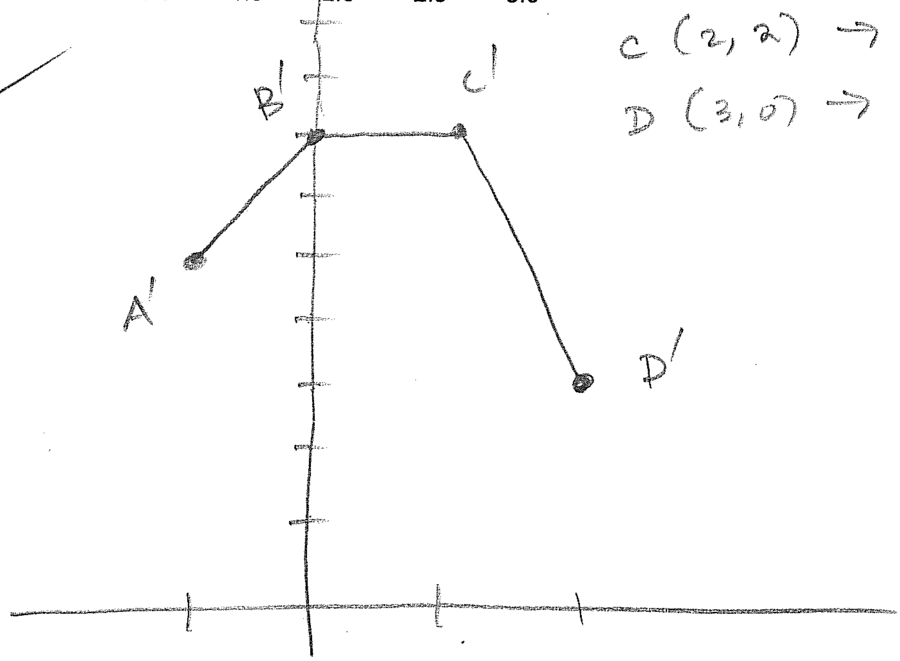

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5. The graph of the function $f(x) = \sqrt{1-x^2}$ is the semi-circle pictured below on the lower left. Use transformations of f to create the function whose graph is given below on the upper right.

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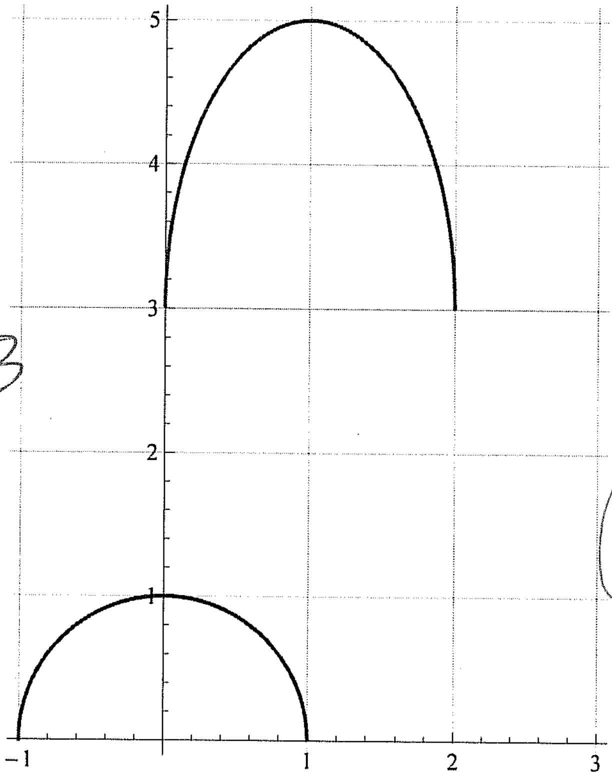

1. shift right by 1
2. scale in y by 2,
3. shift up by 3.

$y = 2 \cdot f(x-1) + 3.$

$y = 2\sqrt{1-(x-1)^2} + 3.$

6. Give a formal proof of the statement $\lim_{x\to 2}(5x-4) = 6$.

5

Prelim: $0 < |x-2| < \delta.$

$|(5x-4) - 6| = |5x-10|$

$= 5|x-2|$

$< 5\delta.$

Let $\delta = \varepsilon/5$

Let $\varepsilon > 0$ be given. Set $\delta = \varepsilon/5$. If $0 < |x-2| < \delta$, then

$|(5x-4)-6| = |5x-10|$

$= 5|x-2|$

$< 5\delta$

$= \varepsilon.$

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3

7. The point $P(1,\cancel{2})$ is on the curve $y = f(x) = 1 + 2x^2$.

(a) Let Q_1 be the point on the graph of f whose x-coordinate is 2. Find the slope of the secant line through P and Q_1. Sketch the corresponding secant line below.

$$Q_1\left(2,\ 1+2(2)^2\right) = (2, 9)$$

3

$$m_{PQ_1} = \frac{9-3}{2-1} = \frac{6}{1} = 6$$

(b) Let Q_2 be the point whose x-coordinate is 1.2. Find the slope of the secant line through P and Q_2. Sketch the corresponding secant line below.

$$Q_2\left(1.2,\ 1+2(1.2)^2\right) = (1.2,\ 3.88)$$

3

$$m_{PQ_2} = \frac{3.88-3}{1.2-2} = 4.4$$

(c) Sketch the tangent line to the graph of f at P.

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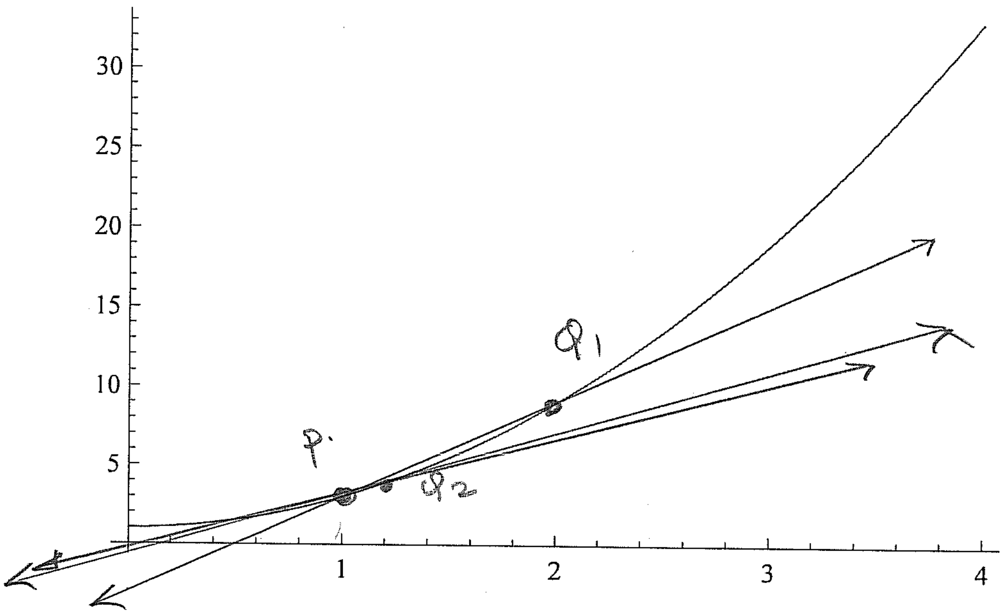

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8. Evaluate the following limits. If the limit does not exist, explain this.

(a) $\lim_{x\to 2} \frac{x^2-x-3}{x^2-4}$

$(x-2)(x+2)$

3

$\lim_{x\to 2^+} \frac{x^2-x-3}{x^2-4} = -\infty$

$\lim_{x\to 2^-} \frac{x^2-x-3}{x^2-4} = +\infty$

Since these do not agree, the limit d.n.e.

−2 0 2

+ + | − − − − | + + + +

(b) $\lim_{x\to 2} \frac{x^3-8}{x^2-4} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x+2)} = \frac{12}{4} = 3.$

3

2	1	0	0	−8
		2	4	8
	1	2	4	0

(c) $\lim_{t\to 9} \frac{3-\sqrt{t}}{t-9} = \lim_{t\to 9} \frac{(3-\sqrt{t})(3+\sqrt{t})}{(t-9)(3+\sqrt{t})} = \lim_{t\to 9} \frac{9-t}{(t-9)(3+\sqrt{t})}$

3

$= \lim_{t\to 9} \frac{-1}{3+\sqrt{t}} = -\frac{1}{6}.$

9. Let $f(s) = \frac{s^2+s}{|s|}$

(a) Rewrite f without using absolute values. Simplify your expression.

$|s| = \begin{cases} s & \text{if } s \geq 0 \\ -s & \text{if } s < 0 \end{cases}$

$f(s) = \begin{cases} s+1 & \text{if } s \geq 0 \\ -s-1 & \text{if } s < 0 \end{cases}$

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(b) Show that $\lim_{s\to 0} f(s)$ does not exist.

2

$\lim_{s\to 0^+} f(s) = \lim_{s\to 0^+} (s+1) = 1$

$\lim_{s\to 0^-} f(s) = \lim_{s\to 0^-} (-s-1) = -1$

Since these limits do not agree, the limit d.n.e.