Assignment #9 **Name** Answer Key

4 December 2009

1. A particle moves on a horizontal line in such a way that its position for any time $t \geq 0$ (measured in seconds) is given by $x(t) = t^3 - 6t^2 + 9t$ meters.

   (a) Find the velocity of the particle and make a sign chart for the velocity function.

   $v(t) = \frac{dx}{dt} = 3t^2 - 12t + 9 = 3(t^2 - 4t + 3)$

   $= 3(t-3)(t-1)$

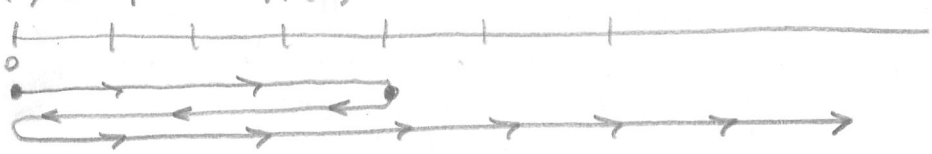
   (b) On what time intervals is the particle moving to the right? On what time intervals is the particle moving to the left?

   right on $[0, 1)$ and $(3, +\infty)$

   left on $(1, 3)$.

   (c) Make a schematic drawing of the motion of the particle (see, for example, Figure 2 on page 172).

   $x(1) = 4$ $x(3) = 0$

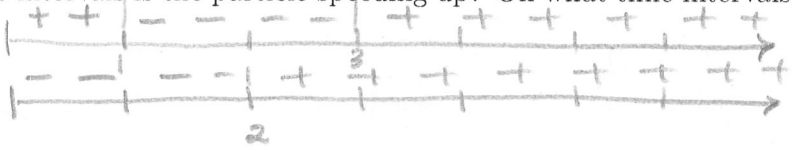
   

   (d) Find the acceleration of the particle. Make a sign chart for the acceleration function.

   $a(t) = 6t - 12 = 6(t-2)$.

   (e) On what time intervals is the particle speeding up? On what time intervals is the particle slowing down?

   slowing down : $[0, 1)$, $(2, 3)$

   speeding up : $(1, 2)$, $(3, +\infty)$.

   (f) Find the displacement and total distance traveled by the particle on the time interval $[0, 4]$.

   $x(4) = 64 - 96 + 36 = 4$

   displacement $= x(4) - x(0) = 4 - 0 = 4$

   total distance $= 4 + 4 + 4 = 12$.

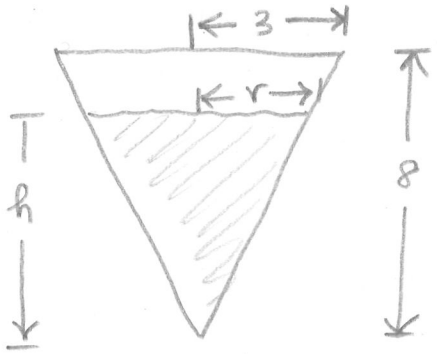
2. A water tank has the shape of an inverted cone (tip down) with a base radius of 3 meters and a height of 8 meters. If water is being added to the tank at a rate of 2 cubic meters per hour, then find the rate at which the water level is rising when the depth of the water is 5 meters. Approach this problem in phases:

   Phase I: Draw a picture to represent a generic point in time; introduce the relevant variables; find the relationship between the volume of the water and its height.

   Phase II: Differentiate with respect to t and solve for the required rate.

   Phase III: Plug in the particular instance in time and find the required rate.

I.



$$\frac{h}{r} = \frac{8}{3} \therefore \quad r = \frac{3}{8}h$$

$$V = \frac{1}{3}\pi\left(\frac{3}{8}h\right)^2 h$$

$$V = \frac{1}{3}\pi\frac{9}{64}h^3 = \frac{3\pi}{64}h^3$$

II. $$\frac{dV}{dt} = \frac{3\pi}{64}\cdot 3h^2\cdot\frac{dh}{dt} \quad \therefore$$

$$\frac{dh}{dt} = \frac{64}{9\pi h^2}\frac{dV}{dt}.$$

III. @ $h = 5$ meters,

$$\frac{dh}{dt} = \frac{64}{9\pi(5)^2}(2)$$

$$\boxed{\frac{dh}{dt} = \frac{128}{225\pi}\ \frac{m}{s}}$$