

Assignment #7

Name Answer Key

Due 6 November 2009

1. In each case, use the rules of differentiation to find y' .

$$(a) y = 2\sqrt{t} + \frac{5t}{t^4 + 3t^2 + 7} = 2t^{1/2} + \frac{5t}{t^4 + 3t^2 + 7}$$

$$y' = 2 \cdot \frac{1}{2} \cdot t^{-1/2} + \frac{(t^4 + 3t^2 + 7)(5) - 5t(4t^3 + 6t)}{(t^4 + 3t^2 + 7)^2}$$

$$(b) y = \frac{3t^2 + 5t - 4}{\sqrt{t}}$$

$$y' = \frac{t^{1/2}(6t + 5) - (3t^2 + 5t - 4)\left(\frac{1}{2}t^{-1/2}\right)}{(\sqrt{t})^2}$$

2. The tangent line to the curve $y = f(x)$ at $P(1, 3)$ is $y = 2x + 1$. The tangent line to the curve $y = g(x)$ at $Q(1, 4)$ is $y = -x + 5$.

(a) What are $f'(1)$ and $g'(1)$?

These values come from the slopes of the tangent lines; thus,

$$f'(1) = 2 \text{ and } g'(1) = -1$$

(b) Let A be the point on the curve $y = f(x)g(x)$ with x -coordinate 1. Find the y -coordinate of A .

$$f(1) = 3 \text{ and } g(1) = 4 \quad \therefore$$

$$y = f(1)g(1) = 3 \cdot 4 = 12 \quad A(1, 12).$$

(c) Find the equation of the tangent line to the curve $y = f(x)g(x)$ at the point A .

$$y' = f'(x)g(x) + f(x)g'(x)$$

$$y'(1) = f'(1)g(1) + f(1)g'(1) = 2(4) + 3(-1) = 5.$$

$$(y - 12) = 5(x - 1) \text{ or } \boxed{y = 5x + 7}$$