Assignment #6 **Name** Answer Key

5 pts.

Due 30 October 2009

1. Let $f(x) = \dfrac{x}{1+x^2}$. Use the definition of the derivative to find $f'(x)$.

$$f(x+h) - f(x) = \frac{x+h}{1+(x+h)^2} - \frac{x}{1+x^2}$$

1.5

$$= \frac{(x+h)(1+x^2) - x\left(1+(x+h)^2\right)}{\left[1+(x+h)^2\right]\left[1+x^2\right]}$$

2.5

$$= \frac{x + x^3 + h + hx^2 - x\left(1+x^2+2xh+h^2\right)}{\left(1+(x+h)^2\right)\left(1+x^2\right)}$$

$$= \frac{x + x^3 + h + hx^2 - x - x^3 - 2x^2h - xh^2}{\left(1+(x+h)^2\right)\left(1+x^2\right)}$$

$$= \frac{h - x^2h - xh^2}{\left(1+(x+h)^2\right)\left(1+x^2\right)}$$

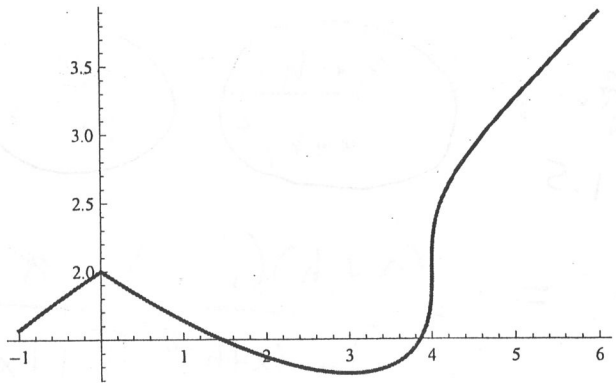
$$\frac{f(x+h)-f(x)}{h} = \frac{1 - x^2 - xh}{\left(1+(x+h)^2\right)\left(1+x^2\right)}$$

5

$$f'(x) = \lim_{h\to 0} \frac{1-x^2-xh}{\left(1+(x+h)^2\right)\left(1+x^2\right)} = \frac{1-x^2}{\left(1+x^2\right)^2}$$

2. The graph of f is given below. State, with reasons, the point(s) at which f is not differentiable. Sketch the derivative of f.

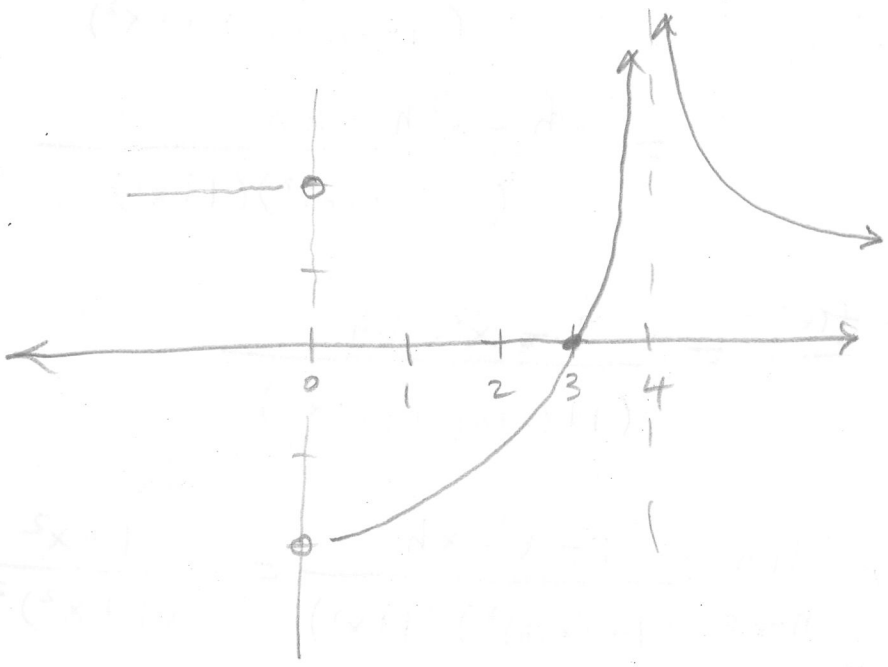
Figure 1: The graph of f



f is not diff @ x=0 ; corner

f is not diff @ x=4 ; vert. tangent line

also notice that f'(3)=0



Graph of f'