§12.3–The Integral Test

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Outline

Theorem

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If $f(x)$ *is continuous, nonnegative, and decreasing on the interval* $[1, \infty)$*, then*

$$\sum_{n=1}^{\infty} f(n) \qquad \textit{and} \qquad \int_1^{\infty} f(x)dx$$

converge or diverge together.

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$$\int_1^{n+1} f(x)dx \leq s_n \leq f(1) + \int_1^n f(x)dx.$$

- If the integral converges, then so does s_n; if s_n converges, so does the integral. □

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- $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k\big(\ln(k)\big)^2}$