§6.1–Integration by Parts

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Outline

The formula

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- Since $D_x\big(f(x)g(x)\big) = f'(x)g(x) + f(x)g'(x)$, we have

$$\int D_x\big(f(x)g(x)\big)\,dx = \int f'(x)g(x)dx + \int f(x)g'(x)dx.$$

This leads us to the formula

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$

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- If $u = f(x)$ and $v = g(x)$, then $du = f'(x)dx$ and $dv = g'(x)dx$ and

$$\int udv = uv - \int vdu,$$

  which is a short form of the *integration by parts formula*.

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- $\int_0^1 xe^x dx$
- $\int e^x \sin(x) dx$

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Show that

$$\int \sin^n(x)dx = -\frac{1}{n}\cos(x)\sin^{n-1}(x) + \frac{n-1}{n}\int \sin^{n-2}(x)dx.$$

This is called a reduction formula: the power of the sine in the integral is reduced from n to $n-2$.

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Solution

Use integration by parts with $u = \sin^{n-1}(x)$ *and* $dv = \sin(x)dx$.

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Problem

Evaluate $\int_0^{\pi} \sin^7(x)dx$ and $\int_0^{\pi} \sin^8(x)dx$.