§5.3–The Natural Exponential Function

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Outline

Definition

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Elementary properties of exp

- The graph of exp can be obtained directly from the graph of ln.
- We have the important inverse relationships:

$$\ln(\exp(x)) = x \quad \text{and} \quad \exp(\ln(x)) = x.$$

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- In particular,

$$\ln e^x = x \quad \text{and} \quad e^{\ln(x)} = x.$$

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Solve the following equations:

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- $e^{2x} + 2e^x - 8 = 0$

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Find y' in each case:

- $y = e^{x^2}$
- $y = x^3 \exp\left(\frac{x+1}{x+2}\right)$
- $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

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- $\displaystyle\int \frac{e^x}{e^x + 1}\,dx$
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Problem

Sketch the graph of $y = xe^{-x}$.