

§5.2–The Natural Logarithm Function

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Outline

- 1 The log base 10
- 2 The natural logarithm
- 3 Further properties of the natural logarithm
- 4 The graph of the log function
- 5 The number e , Euler's number
- 6 Further derivative problems
- 7 Integration
- 8 Logarithmic differentiation

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- $\log_{10}(a^r) = r \log_{10}(a)$, $a > 0$, r rational.

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For $x > 0$, let

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Problem

Show that $.5 \leq \ln(2) \leq 1$.

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Sketch the graph of $\ln(x)$.

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- Since $\ln(e) = 1$, e is called the *base* of the natural logarithm function.

Theorem (The chain rule)

If f is a positive, differentiable function, then

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- $\int_{-3}^{-1} (x+1)^{-1} dx$
- $\int \tan(x) dx$

Problem

Use the logarithm function and its properties to evaluate the derivative of

$$f(x) = \frac{x^2(x-4)^3}{(x^2+1)^4}.$$