Furman University

If – Then Statements

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1. If – Then Statements

An example of an if..then statement is "If I do not understand this review section, then I will read it again."

An if..then statement can be phrased in a variety of ways without changing its meaning. A few examples of how our first example can be rephrased are as follows:

"My not understanding this review section implies that I will read it again."

"My not understanding this review section is a necessary condition for my reading it again."

"I do not understand this review section only if I will read it again."

"I will read this review section again if I don't understand it."

An if..then statement consists of two parts, the "if" part called the hypothesis and the "then" part called the conclusion. Each of these parts have a truth value. The truth values of the hypothesis and the conclusion determine the truth value of the if..then statement. The if..then statement is false only when the hypothesis true but the conclusion is false.

Truth tables are a convenient way to determine the truth value of a logical statement according to

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

When we proof a theorem in class, we are simply showing that the if..then statement is true. To do this we will assume the "if" part is true and attempt to show the "then" part is also true. Since an if..then statement is false only when the hypothesis is true and the conclusion is false, we only need to show that the conclusion is always true when the hypothesis is true.

EXERCISE 1. Make a truth table for the $q \to p$ and compare it with $p \to q$. Are they the same or different?

EXERCISE 2. Compare the statement $p \to q$ and $\tilde{q} \to \tilde{p}$. Here $\tilde{p}$ stands for the negation of p. It has the opposite truth value of that of p.

EXERCISE 3. How do the statements "p whenever q" and "p only if q" compare with "if p then q." Also where does "p if and only if q" fit in?

Companion to Calculus: Page 128, 1 - 4.

Solutions to Exercises

p	q	$p \to q$	$q \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Exercise 1. We can see that they aren't the same. Thus a statement $p \to q$ might be true but its converse $q \to p$ false. An example of this is the statement " If $f(x)$ is differentiable on (a, b), then it is continuous on (a, b). This statment is true but its converse "If $f(x)$ is continuous on (a, b) then it is differentiable on (a, b) is **false**.

Exercise 1

p	q	$p \to q$	$\tilde{q} \to \tilde{p}$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Exercise 2. We see that these are the same. The statement $\tilde{q} \to \tilde{p}$ is called the *contrapositive* of $p \to q$.

Exercise 2

Exercise 3. "p whenever q" means the same thing as "$q \to p$.", and "p only if q" means the same thing as "$p \to q$." The statement "p if and only if q" means $p \to q$ AND $q \to p$. This statement is often abbreviated "iff" and perhaps written $p \leftrightarrow q$.

Exercise 3