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# PATHS AND CIRCUITS IN $\mathbb{G}$-GRAPHS OF CERTAIN NON-ABELIAN GROUPS

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Abstract. In [BJRTD08], necessary and sufficient conditions were given for the existence of Eulerian and Hamiltonian paths and circuits in the $\mathbb{G}$-graph of the dihedral group $D_n$. In this paper, we consider the $\mathbb{G}$-graphs of the quasihedral, modular, and generalized quaternion group. These groups are of rank 2 and we consider only the graphs $\Gamma(G, S)$ where $|S| = 2$.

## 1. Introduction

Let $G$ be a finitely generated group with generating set $S = \{s_1, \cdots, s_k\}$. For a subgroup $H$ of G, define the subset $T_H$ of $G$ to be a *left transversal* for $H$ if $\{xH \mid x \in T_H\}$ is precisely the set of all left cosets of $H$ in $G$. For each $s_i \in S$ let $H_i = \langle s_i \rangle$. Associate a simple graph $\Gamma(G, S)$ to $(G, S)$ with vertex set $V = \{x_j H_i \mid x_j \in T_{H_i}\}$. Two distinct vertices $x_j H_i$ and $x_l H_k$ in $V$ are joined by an edge if $x_j\langle s_i\rangle \cap x_l\langle s_k\rangle$ is nonempty. The edge set $E$ consists of pairs $(x_j H_i, x_l H_k)$. $\Gamma(G, S)$ defined this way has no multiedge or loop. A multiedge graph was defined similarly in 2004. Many of the results about this graph [[BG04], [BGL05], [BG05], and [BG07]] can be modified for the simple graph, $\Gamma(G, S)$, [D08]. The main object of this paper is to study the existence of Eulerian and Hamiltonian paths and circuits in the $\mathbb{G}$-graphs of the quasihedral, modular, and generalized quaternion group. To explore the existence of Eulerian paths and circuits in $\Gamma(G, S)$, we recall a few theorems of Euler and a result from [BJRTD08].

**Theorem 1.** *(Euler) Let $\Gamma$ be a nontrivial connected graph. Then $\Gamma$ has an Eulerian circuit if and only if every vertex is of even degree.*

**Theorem 2.** *(Euler) Let $\Gamma$ be a nontrivial connected graph. Then $\Gamma$ has an Eulerian path if and only if $\Gamma$ has exactly two vertices of odd degree. Furthermore, the path begins at one of the vertices of odd degree and terminates at the other.*

**Lemma 3.** [BJRTD08] *If $G$ is a group with generating set $S = \{s_1, \cdots, s_n\}$ and $S_{i,j} = |\langle s_i\rangle \cap \langle s_j\rangle|$, then the degree of the vertex $\langle s_i\rangle$, denoted deg($\langle s_i\rangle$), is*

$$deg(\langle s_i\rangle) = \left(\sum_{j=1}^{n} |s_i|/S_{i,j}\right) - 1.$$

*Remark* 1. Notice that $deg(\langle s_i\rangle) = deg(x_j\langle s_i\rangle)$ for all $x_j\langle s_i\rangle$ in $V_i$.

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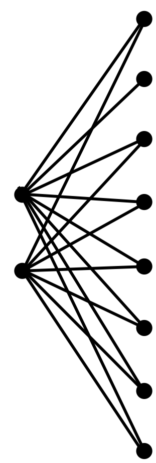
We consider the $\mathbb{G}$-graphs of the quasihedral, modular, and generalized quaternion group. We start with a few examples of the graphs.

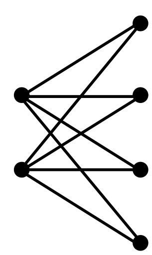
*Example* 1.

(i) The modular group, $M$, has presentation $\langle s, t \mid s^8 = t^2 = e, st = ts^5 \rangle$. Letting $S = \{s, t\}$, the $\mathbb{G}$ graph of this group is $\Gamma(M, S)$.



$K_{2,8}$

(ii) The quasihedral group, $QS$, has presentation $\langle s, t \mid s^8 = t^2 = e, st = ts^3 \rangle$. Letting $S = \{s, ts\}$, the $\mathbb{G}$ graph of this group is $\Gamma(QS, S)$.

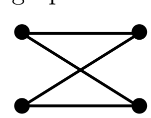


$K_{2,4}$

(iii) The generalized quaternion group, $Q_{2^n}$, has presentation

$$\langle s, t \mid s^{2^{n-1}} = e, s^{2^{n-2}} = t^2, tst^{-1} = s^{-1} \rangle.$$

Letting $n = 3, S = \{s, t\}$, the $\mathbb{G}$ graph of this group is $\Gamma(Q_{2^3}, S)$.



$K_{2,2}$

The next lemma pertains to all of the groups in question.

**Lemma 4.** *Let $G = M, QS$, or $Q_{2^n}$ and let $j$ be an odd integer then*

$$\langle s^j \rangle = \langle s \rangle = \{s, s^2, \cdots, s^{|s|-1}, e\}.$$

*Proof.* For each of the above groups, $|s|$ is even. So $\gcd(j, |s|) = 1$ and there exist $x, y \in \mathbb{Z}$ such that $jx + |s|y = 1$. So

$$\begin{aligned} s^1 &= s^{jx+|s|y} \\ s^1 &= s^{jx}s^{|s|y} \\ s^1 &= (s^j)^x(s^{|s|})^y \\ s^1 &= (s^j)^x(e)^y \end{aligned}$$

Therefore $s^1 = (s^j)^x$ and $\langle s\rangle = \langle s^j\rangle$.

□

## 2. The Modular group

Recall that the modular group, $M$, has presentation $\langle s, t \mid s^8 = t^2 = e, st = ts^5\rangle$. Next we determine the existence of Eulerian and Hamiltonian circuits and paths.

**Lemma 5.** *If $G$ is the modular group and $n$ is odd, then*

$$\langle ts^n\rangle = \langle ts\rangle = \{ts, s^6, ts^7, s^4, ts^5, s^2, ts^3, e\}.$$

**Lemma 6.** *If $G$ is the modular group, then $\langle ts^2\rangle = \langle ts^6\rangle = \{ts^2, s^4, ts^6, e\}$.*

**Lemma 7.** *If $G$ is the modular group and $n = 2$ or $6$, then $|\langle s\rangle \cap \langle ts^n\rangle| = 2$.*

**Lemma 8.** *If $G$ is the modular group and $n$ is odd, then $|\langle s\rangle \cap \langle ts^n\rangle| = 4$.*

**Theorem 9.** *If $G$ is the modular group, and $S$ is a minimal generating set, then $\Gamma(G, S)$ contains an Eulerian circuit.*

*Proof.* Let $G$ be the modular group and $S$ be a minimal generating set. Then $S = \{s^n, ts^k\}$, where $n$ is odd, $1 \le n \le 7$, and $0 \le k \le 7$ or $S = \{ts^n, ts^m\}$, where $n$ is odd and $m$ is even. By using the lemmas above there exists three distinct graphs.

case i) Let $S = \{s^n, t\}$ where $n$ is odd, then $S_{1,2} = S_{2,1} = |\langle s^n\rangle \cap \langle t\rangle| = 1$ and $deg(\langle s^n\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_1\rangle|}{S_{1,j}}\right) - 1 = \frac{8}{S_{1,1}} + \frac{8}{S_{1,2}} - 1 = \frac{8}{8} + \frac{8}{1} - 1 = 8$, which is even. Similarly $deg(\langle t\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_2\rangle|}{S_{2,j}}\right) - 1 = \frac{2}{S_{2,1}} + \frac{2}{S_{2,2}} - 1 = \frac{2}{1} + \frac{2}{2} - 1 = 2$, which is even. This graph is $K_{2,8}$ and contains an Eulerian circuit.

case ii) Let $S = \{s^n, ts^m\}$ where $n, m$ are odd, then $S_{1,2} = S_{2,1} = |\langle s^n\rangle \cap \langle ts^m\rangle| = 4$ and $deg(\langle s^n\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_1\rangle|}{S_{1,j}}\right) - 1 = \frac{8}{S_{1,1}} + \frac{8}{S_{1,2}} - 1 = \frac{8}{8} + \frac{8}{4} - 1 = 2$, which is even. Similarly $deg(\langle ts^m\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_2\rangle|}{S_{2,j}}\right) - 1 = \frac{8}{S_{2,1}} + \frac{8}{S_{2,2}} - 1 = \frac{8}{4} + \frac{8}{8} - 1 = 2$, which is even. This graph is $K_{2,2}$ and contains an Eulerian circuit.

case iii) Let $S = \{s^n, ts^k\}$ where $n$ is odd and $k = 2$ or $6$, then $S_{1,2} = S_{2,1} = |\langle s^n\rangle \cap \langle ts^k\rangle| = 2$ and $deg(\langle s^n\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_1\rangle|}{S_{1,j}}\right) - 1 = \frac{8}{S_{1,1}} + \frac{8}{S_{1,2}} - 1 = \frac{8}{8} + \frac{8}{2} - 1 = 4$, which is even. Similarly $deg(\langle ts^k\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_2\rangle|}{S_{2,j}}\right) - 1 = \frac{2}{S_{2,1}} + \frac{2}{S_{2,2}} - 1 = \frac{4}{2} + \frac{4}{4} - 1 = 2$, which is even. This graph is $K_{2,4}$ and contains an Eulerian circuit.

case iv) Let $S = \{s^n, ts^4\}$ where $n$ is odd, then $S_{1,2} = S_{2,1} = |\langle s^n\rangle \cap \langle ts^4\rangle| = 1$ and $deg(\langle s^n\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_1\rangle|}{S_{1,j}}\right) - 1 = \frac{8}{S_{1,1}} + \frac{8}{S_{1,2}} - 1 = \frac{8}{8} + \frac{8}{1} - 1 = 8$, which is even. Similarly $deg(\langle ts^4\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_2\rangle|}{S_{2,j}}\right) - 1 = \frac{2}{S_{2,1}} + \frac{2}{S_{2,2}} - 1 = \frac{2}{1} + \frac{2}{2} - 1 = 2$, which is even. This graph is $K_{2,8}$ and contains an Eulerian circuit.

case v) Let $S = \{ts^n, t\}$ where $n$ is odd, then $S_{1,2} = S_{2,1} = |\langle ts^n\rangle \cap \langle t\rangle| = 1$ and $deg(\langle ts^n\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_1\rangle|}{S_{1,j}}\right) - 1 = \frac{8}{S_{1,1}} + \frac{8}{S_{1,2}} - 1 = \frac{8}{8} + \frac{8}{1} - 1 = 8$, which is even. Similarly $deg(\langle t\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_2\rangle|}{S_{2,j}}\right) - 1 = \frac{2}{S_{2,1}} + \frac{2}{S_{2,2}} - 1 = \frac{2}{1} + \frac{2}{2} - 1 = 2$, which is even. This graph is $K_{2,8}$ and contains an Eulerian circuit.

case vi) Let $S = \{ts^n, ts^k\}$ where $n$ is odd and $k = 2$ or $6$, then $S_{1,2} = S_{2,1} = |\langle ts^n\rangle \cap \langle ts^k\rangle| = 2$ and $deg(\langle ts^n\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_1\rangle|}{S_{1,j}}\right) - 1 = \frac{8}{S_{1,1}} + \frac{8}{S_{1,2}} - 1 = \frac{8}{8} + \frac{8}{2} - 1 = 4$, which is even. Similarly $deg(\langle ts^k\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_2\rangle|}{S_{2,j}}\right) - 1 = \frac{2}{S_{2,1}} + \frac{2}{S_{2,2}} - 1 = \frac{4}{2} + \frac{4}{4} - 1 = 2$, which is even. This graph is $K_{2,4}$ and contains an Eulerian circuit.

case vii) Let $S = \{ts^n, ts^4\}$ where $n$ is odd, then $S_{1,2} = S_{2,1} = |\langle ts^n\rangle \cap \langle ts^4\rangle| = 1$ and $deg(\langle ts^n\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_1\rangle|}{S_{1,j}}\right) - 1 = \frac{8}{S_{1,1}} + \frac{8}{S_{1,2}} - 1 = \frac{8}{8} + \frac{8}{1} - 1 = 8$, which is even. Similarly $deg(\langle ts^4\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_2\rangle|}{S_{2,j}}\right) - 1 = \frac{2}{S_{2,1}} + \frac{2}{S_{2,2}} - 1 = \frac{2}{1} + \frac{2}{2} - 1 = 2$, which is even. This graph is $K_{2,8}$ and contains an Eulerian circuit.

□

*Remark* 2. For all minimal generating sets, $\Gamma(M,S)$ does not contain an Eulerian path.

**Theorem 10.** *If $G$ is the modular group, and $S = \{s^n, ts^m\}$ where $n, m$ are odd, then $\Gamma(G,S)$ contains a Hamiltonian circuit and a Hamiltonian path.*

*Proof.* The vertex set of $\Gamma(M,S)$ is $V(\Gamma(M,S)) = \{\langle s^n\rangle, t\langle s^n\rangle, \langle ts^m\rangle, t\langle ts^m\rangle\}$ . A Hamiltonian circuit is given by

$$\langle s^n\rangle, \langle ts^m\rangle, t\langle s^n\rangle, t\langle ts^m\rangle, \langle s^n\rangle.$$

A Hamiltonian path is given by

$$\langle s^n\rangle, \langle ts^m\rangle, t\langle s^n\rangle, t\langle ts^m\rangle.$$

□

*Remark* 3. $S = \{s^n, ts^m\}$ where $n, m$ are odd is the only minimal generating set of $M$ that yields a graph that contains a Hamiltonian circuit (path).

## 3. The Quasihedral group

Recall that the quasihedral group, $QS$, has presentation $\langle s, t \mid s^8 = t^2 = e, st = ts^3\rangle$. Next we determine the existence of Eulerian and Hamiltonian circuits and paths.

**Lemma 11.** *If $G$ is the quasihedral group and $n$ is 1 or 5, then $\langle ts^n\rangle = \{ts, s^4, ts^5, e\}$.*

**Lemma 12.** *If $G$ is the quasihedral group and $n$ is 3 or 7, then $\langle ts^n\rangle = \{ts^3, s^4, ts^7, e\}$.*

**Lemma 13.** *If $G$ is the quasihedral group and $n$ is even, then $\langle ts^n\rangle = \{ts^n, e\}$.*

**Lemma 14.** *If $G$ is the quasihedral group and $n$ is even, then $|\langle s\rangle \cap \langle ts^n\rangle| = 1$.*

**Lemma 15.** *If $G$ is the quasihedral group and $n$ is odd, then $|\langle s\rangle \cap \langle ts^n\rangle| = 2$.*

**Theorem 16.** *If $G$ is the quasihedral group, and $S$ is a minimal generating set, then $\Gamma(G, S)$ contains a Eulerian circuit.*

*Proof.* Let $G$ be the quasihedral group and $S$ be a minimal generating set. Then $S = \{s^n, ts^k\}$, where $n$ is odd and $1 \leq n \leq 7$ and $1 \leq k \leq 3$ or $S = \{ts^n, ts^m\}$, where $n$ is odd and $m$ is even. By using the above lemmas, there exists three distinct graphs.

case i) Let $S = \{s^n, ts^m\}$, where $n, m$ are odd, then $S_{1,2} = S_{2,1} = |\langle s^n\rangle \cap \langle ts^m\rangle| = 2$ and $deg(\langle s^n\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_1\rangle|}{S_{1,j}}\right) - 1 = \frac{8}{S_{1,1}} + \frac{8}{S_{1,2}} - 1 = \frac{8}{8} + \frac{8}{2} - 1 = 4$, which is even. Similarly, $deg(\langle ts^m\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_2\rangle|}{S_{2,j}}\right) - 1 = \frac{4}{S_{2,1}} + \frac{4}{S_{2,2}} - 1 = \frac{4}{2} + \frac{4}{4} - 1 = 2$, which is even. This graph is $K_{2,4}$ and contains an Eulerian circuit.

case ii) Let $S = \{s^n, ts^m\}$, where $n$ is odd and $m$ is even, then $S_{1,2} = S_{2,1} = |\langle s^n\rangle \cap \langle ts^m\rangle| = 1$ and $deg(\langle s^n\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_1\rangle|}{S_{1,j}}\right) - 1 = \frac{8}{S_{1,1}} + \frac{8}{S_{1,2}} - 1 = \frac{8}{8} + \frac{8}{1} - 1 = 8$, which is even. Similarly, $deg(\langle ts^m\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_2\rangle|}{S_{2,j}}\right) - 1 = \frac{2}{S_{2,1}} + \frac{2}{S_{2,2}} - 1 = \frac{2}{1} + \frac{2}{2} - 1 = 2$, which is even. This graph is $K_{2,8}$ and contains an Eulerian circuit.

case iii) Let $S = \{ts^n, ts^m\}$ where $n$ is odd and $m$ is even, then $S_{1,2} = S_{2,1} = |\langle ts^n\rangle \cap \langle ts^m\rangle| = 1$ and $deg(\langle ts^n\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_1\rangle|}{S_{1,j}}\right) - 1 = \frac{4}{S_{1,1}} + \frac{4}{S_{1,2}} - 1 = \frac{4}{4} + \frac{4}{1} - 1 = 4$, which is even. Similarly $deg(\langle ts^m\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_2\rangle|}{S_{2,j}}\right) - 1 = \frac{2}{S_{2,1}} + \frac{2}{S_{2,2}} - 1 = \frac{2}{1} + \frac{2}{2} - 1 = 2$, which is even. By applying Euler's theorem, this graph contains an Eulerian circuit.

□

*Remark* 4. For all minimal generating sets, $\Gamma(QS, S)$ does not contain an Eulerian path, a Hamiltonian path, or a Hamiltonian circuit.

## 4. Generalized Quaternion Group

Recall that the generalized quaternion group, $Q_{2^n}$, has presentation $\langle s, t \mid s^{2^{n-1}} = e, s^{2^{n-2}} = t^2, tst^{-1} = s^{-1}\rangle$. Next we determine the existence of Eulerian and Hamiltonian circuits and paths.

**Lemma 17.** *If $G$ is the generalized quaternion group, then $t^4 = e$.*

*Proof.* Let $G$ be the generalized quaternion group. Recall that $t^2 = s^{2^{n-2}}$. Squaring both sides,

$$(t^2 = s^{2^{n-2}})^2$$
$$t^4 = s^{2^{n-1}} = e.$$

□

**Lemma 18.** *Let $G$ be the generalized quaternion group, then $(ts^j)^2 = t^2$ for all $j$.*

*Proof.* We proceed with induction on $j$. Let $j = 1$, then $(ts^1)^2 = tsts = ts(s^{-1}t) = t^2$ and the theorem holds for $j = 1$. Assume that the theorem holds for $j = k$, i.e, $(ts^k)^2 = t^2$.

Now let $j = k+1$, then $(ts^{k+1})^2 = ts^{k+1}ts^{k+1} = ts^{k+1}tss^k = ts^{k+1}s^{-1}ts^k = ts^kts^k = (ts^k)^2 = t^2$. Therefore $(ts^j)^2 = t^2$ for all $j$.

□

**Lemma 19.** *Let $G$ be the generalized quaternion group, then $\langle ts^j\rangle = \{ts^j, t^2, t^3s^j, e\}$ for all $j$.*

**Lemma 20.** *If $G$ is the generalized quaternion group and $\langle ts^j\rangle \neq \langle ts^k\rangle$, then $\langle ts^j\rangle \cap \langle ts^k\rangle = \{t^2, e\}$ and $|\langle ts^j\rangle \cap \langle ts^k\rangle| = 2$.*

**Theorem 21.** *If $G$ is the generalized quaternion group, and $S$ is a minimal generating set, then $\Gamma(G,S)$ contains an Eulerian circuit.*

*Proof.* Let $G$ be the generalized quaternion group and $S$ be a minimal generating set. Then, $S = \{s^k, ts^j\}$ where $k$ is odd or $S = \{ts^k, ts^m\}$, where $k$ is odd and $m$ is even.

case i) Let $S = \{s^k, ts^j\}$ where $k$ is odd, then $S_{1,2} = S_{2,1} = |\langle s^k\rangle \cap \langle ts^j\rangle| = 2$ and $deg(\langle s^k\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_1\rangle|}{S_{1,j}}\right) - 1 = \frac{2^{n-1}}{S_{1,1}} + \frac{2^{n-1}}{S_{1,2}} - 1 = \frac{2^{n-1}}{2^{n-1}} + \frac{2^{n-1}}{2} - 1 = \frac{2^{n-1}}{2} = 2^{n-2}$, which is even. Similarly $deg(\langle ts^j\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_2\rangle|}{S_{2,j}}\right) - 1 = \frac{4}{S_{2,1}} + \frac{4}{S_{2,2}} - 1 = \frac{4}{4} + \frac{4}{2} - 1 = 2$, which is even. This graph is $K_{2,2^{n-2}}$ and contains an Eulerian circuit.

case ii) Let $S = \{ts^k, ts^m\}$, where $k$ is odd and $m$ is even, then $S_{1,2} = S_{2,1} = |\langle ts^k\rangle \cap \langle ts^m\rangle| = 2$ and $deg(\langle ts^k\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_1\rangle|}{S_{1,j}}\right) - 1 = \frac{4}{S_{1,1}} + \frac{4}{S_{1,2}} - 1 = \frac{4}{4} + \frac{4}{2} - 1 = 2$, which is even. Similarly $deg(\langle ts^m\rangle) = \left(\sum_{j=1}^{2} \frac{|\langle s_2\rangle|}{S_{2,j}}\right) - 1 = \frac{4}{S_{2,1}} + \frac{4}{S_{2,2}} - 1 = \frac{4}{4} + \frac{4}{2} - 1 = 2$, which is even. By applying Euler's theorem, this graph contains an Eulerian circuit.

□

*Remark* 5. For all minimal generating sets, $\Gamma(Q_{2^n}, S)$ does not contain an Eulerian path.

**Theorem 22.** *If $G$ is the generalized quaternion group, $Q_{2^n}$, and $S = \{s^k, ts^m\}$ where $k$ is odd, then $\Gamma(G,S)$ contains a Hamiltonian circuit and a Hamiltonian path for $n = 3$.*

*Proof.* The vertex set of $\Gamma(Q_{2^3}, S)$ is $V(\Gamma(Q_{2^3}, S)) = \{\langle s^k\rangle, t\langle s^k\rangle, \langle ts^m\rangle, t\langle ts^m\rangle\}$ . A Hamiltonian circuit is given by

$$\langle s^k\rangle, \langle ts^m\rangle, t\langle s^k\rangle, t\langle ts^m\rangle, \langle s^k\rangle.$$

A Hamiltonian path is given by

$$\langle s^k\rangle, \langle ts^m\rangle, t\langle s^k\rangle, t\langle ts^m\rangle.$$

□

**Theorem 23.** *If $G$ is the generalized quaternion group, $Q_{2^n}$, and $S = \{ts^k, ts^m\}$, where $k$ is odd and $m$ is even, then $\Gamma(G,S)$ contains a Hamiltonian circuit and a Hamiltonian path.*

*Proof.* The vertex set of $\Gamma(Q_{2^n}, S)$ is

$$V(\Gamma(Q_{2^n}, S)) = \{\langle ts^k\rangle, s\langle ts^k\rangle, \cdots, s^{2^{n-2}-1}\langle ts^k\rangle, \langle ts^m\rangle, s\langle ts^m\rangle, \cdots, s^{2^{n-2}-1}\ \langle ts^m\rangle\}.$$

A Hamiltonian circuit is given by

$$\langle ts^k\rangle, \langle ts^m\rangle, s^{k-m}\langle ts^k\rangle, s^{k-m}\langle ts^m\rangle, \cdots, s^{k-(2^{n-2}-1)m}\langle ts^k\rangle, s^{k-(2^{n-2}-1)m}\langle ts^m\rangle, \langle ts^k\rangle.$$

A Hamiltonian path is given by

$$\langle ts^k\rangle, \langle ts^m\rangle, s^{k-m}\langle ts^k\rangle, s^{k-m}\langle ts^m\rangle, s^{k-2m}\langle ts^k\rangle, s^{k-2m}\langle ts^m\rangle, \cdots, \\ s^{k-(2^{n-2}-1)m}\langle ts^k\rangle, s^{k-(2^{n-2}-1)m}\langle ts^m\rangle.$$

□

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