Mathematics 460 Written Assignment #4 DUE: 10-2-17

This is a "class written assignment." Everyone in the class should meet at the regular class time on Friday, September 29, and solve this problem together. Produce a single write-up in LaTeX and turn it in on Monday, October 2. Everyone gets the same score with a maximum of 20 points. This score will be added to your point total on Test 1.

If G is a group (as usual the binary operation will be written multiplicatively), a is any element of G, and S is any subset of G, then Sa and aS are the subsets of G defined by

$$Sa = \{xa \,|\, x \in S\} \quad \text{and} \quad aS = \{ax \,|\, x \in S\}\,.$$

Let H be a subgroup of G and let R be the relation on G defined as follows. For $a, b \in G$,

$$aRb \quad \text{if and only if} \quad ab^{-1} \in H\,.$$

1. Prove that R is an equivalence relation.

2. For $c \in G$, $[c]_R$ denotes the equivalence class of c under the equivalence relation R defined above. Prove that $[a]_R = Ha$, for each $a \in G$.

3. Prove that for all $a \in G$ and all $b \in G$, the equivalence classes $[a]_R$ and $[b]_R$ have the same cardinality. (Use part 2 above.)

4. Now assume that G is a finite group of order n and that H is a subgroup of G. Use parts 2 and 3 above and properties of equivalence classes of an equivalence relation on a set to show that $|G|$ is an (integer) multiple of $|H|$.

5. Let G be the group $\mathbb{Z}_{101}^*$ under the binary operation multiplication modulo 101. Using just part 4 above without doing any other calculations in the group G, make a list of the **possible** orders of subgroups of G.