Mathematics 460 Written Assignment #1 DUE: 8-28-17

Below are two examples of the kind of mathematical writing you will be doing during this term. These two items are slight modifications from the textbook; the first is from page 59, and the second one is from page 105. Your assignment is to use LaTeX to create a PDF file that is a replica of what is below. If you are not able to reproduce these exactly, then submit your best approximation. Please use the files found from the "Documents" tab on the course homepage to discover how to create various parts of the two items.

To submit your solution to this assignment send me an email with both the tex file and a PDF produced from your tex file attached to the email.

Theorem. Every cyclic group is abelian.

Proof. Let G be a cyclic group and let a be a generator of G so that

$$G = \langle a \rangle = \{a^n \colon n \in \mathbb{Z}\}\,.$$

If g_1 and g_2 are any two elements of G, there exist integers r and s such that $g_1 = a^r$ and $g_2 = a^s$. Then

$$g_1 g_2 = a^r a^s = a^{r+s} = a^{s+r} = a^s a^r = g_2 g_1\,,$$

so G is abelian. ■

Example Consider the group $\mathbb{Z}_2 \times \mathbb{Z}_3$, which has $2 \cdot 3 = 6$ elements, namely $(0,0), (0,1), (0,2), (1,0), (1,1)$, and $(1,2)$. We claim that $\mathbb{Z}_2 \times \mathbb{Z}_3$ is cyclic. It is only necessary to find a generator. Let us try $(1,1)$. Here the operations in $\mathbb{Z}_2$ and $\mathbb{Z}_3$ are written additively, so we do the same in the direct product $\mathbb{Z}_2 \times \mathbb{Z}_3$.

$$\begin{aligned}
(1,1) &= (1,1)\\
2(1,1) &= (1,1)+(1,1) = (0,2)\\
3(1,1) &= (1,1)+(1,1)+(1,1) = (1,0)\\
4(1,1) &= 3(1,1)+(1,1) = (1,0)+(1,1) = (0,1)\\
5(1,1) &= 4(1,1)+(1,1) = (0,1)+(1,1) = (1,2)\\
6(1,1) &= 5(1,1)+(1,1) = (1,2)+(1,1) = (0,0)
\end{aligned}$$