MTH-260: Transition to Higher Mathematics
Spring Semester 2018

Prof. Rall

Group #2 Portfolio Problems

Your final portfolio must have complete solutions of each of the three problems in this group. You may submit a write up of your solution one time for review following the directions written in the Guidelines for the Portfolio Project document. Your second submission of a solution will be considered the final submission for that problem. Your score for that problem will be assigned based only on the final submission.

- The deadline for discussion and submission of problems from Group #2 for review is Friday, March 23, 2018.
- The last day to submit problems from Group #2 for a grade is Wednesday, April 4, 2018.

Problem #1

In solving this problem you will probably make use of the proposition near the top of page 106 in the text. If you do, make sure you document its use. Refer to Problem #2 in Group #1 for the definition of Pythagorean triple.

Is the following conjecture true or false? If it is true, call it a proposition and write a proof. If it is false, you must give a specific counterexample to the conjecture.

Conjecture. If (x, y, z) is a Pythagorean triple, then either 3 divides exactly one of the numbers x, y or z or 3 divides all of the numbers x, y and z.

You might be able to use the following lemma. If you do, then you must provide its proof and indicate where you use it.

Lemma. If a is an integer and $a \not\equiv 0 \pmod 3$, then $a^2 \equiv 1 \pmod 3$.

Problem #2

Is the following conjecture true or false? If it is true, call it a proposition and write a proof. If it is false, you must give a specific counterexample to the conjecture.

Conjecture. If A and B are any two sets, then $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$.

Problem #3

Use induction to prove that for every positive integer n,

$$\prod_{k=1}^{n} \left(1 - \frac{1}{2^k}\right) \geq \frac{1}{4} + \frac{1}{2^{n+1}}\,.$$