MTH-260: Transition to Higher Mathematics
Spring Semester 2018

Prof. Rall

Group #1 Portfolio Problems

Your final portfolio must have complete solutions of each of the three problems in this group. You may submit a write up of your solution two times for review following the directions written in the Guidelines for the Portfolio Project document. Your third submission of a solution will be considered the final submission for that problem. Your score for that problem will be assigned based only on the final submission.

- The deadline for discussion and submission of problems from Group #1 for review is February 21, 2018.
- The last day to submit problems from Group #1 for a grade is February 28, 2018.

Problem #1 An integer r is an **odd** integer if there exists an integer k such that $r = 2k + 1$. For two integers m and n we say that m **divides** n provided there is an integer q such that $n = mq$.
Is the following conjecture true or false? If it is true, call it a proposition and write a proof. If it is false, you must give a specific counterexample to the conjecture.

Conjecture. If a and b are odd integers, then 8 divides $a^2 - b^2$.

Problem #2 A triple (a, b, c) of positive integers is called a **Pythagorean triple** if $a^2+b^2 = c^2$. For example, $(5, 12, 13)$ is a Pythagorean triple because $5^2 + 12^2 = 25 + 144 = 169 = 13^2$ shows that $5^2 + 12^2 = 13^2$. It is straightforward to verify that each of $(3, 4, 5), (18, 24, 30), (10, 24, 26)$ and $(8, 15, 17)$ is a Pythagorean triple.

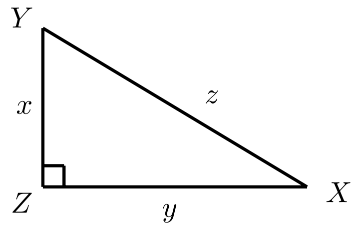
Determine all Pythagorean triples of the form $(n, n+79, n+81)$. A solution to this problem must include a statement of a proposition and a formal proof of that proposition. The proposition must be stated as a conditional statement. It could start out something like this

If n is a positive integer and $(n, n + 79, n + 81)$ is a Pythagorean triple, then ....

Problem #3 This problem involves right triangles that are also isosceles. Your solution to this problem is to write a proof of the proposition written below. The following information must be included at the beginning of your document for this problem.

- The definition of an isosceles triangle.
- A complete statement of the Pythagorean theorem for right triangles.
- The standard formula for the area of an arbitrary triangle and how this formula applies to right triangles.

Also, you should include the following diagram of a right triangle as part of the proof of the proposition. If you download the .tex file associated with this PDF, then you can copy and paste the commands that create this figure.



Proposition. A right triangle is an isosceles triangle if and only if its area is equal to one-fourth times the length of the hypotenuse squared.