Mathematics 150: Summary - Infinite Limits

Here are solutions to the example problems:

1. $\displaystyle\lim_{t\to 4^-} \frac{t^2-2t}{4-t}$

$\displaystyle\lim_{t\to 4^-}(t^2-2t) = 4^2-8=8$, and $\displaystyle\lim_{t\to 4^-}(4-t)=0$. In addition we see that for $t<4$, $0<4-t$. Thus by Case 1 it follows that $\dfrac{t^2-2t}{4-t}$ increases without bound as t approaches 4 from the left. Equivalently, $\displaystyle\lim_{t\to 4^-}\frac{t^2-2t}{4-t}=\infty$.

2. $\displaystyle\lim_{x\to \pi^+} \frac{x^2-3x+2}{\sin(x)}$

From the definition of the sine function we see that $\displaystyle\lim_{x\to\pi^+}\sin(x)=0$ and that for $x>\pi$ (but x close enough to π), $\sin(x)<0$. Considering the numerator we see that $\displaystyle\lim_{x\to\pi^+}(x^2-3x+2)=\pi^2-3\pi+2>0$. We evaluate this limit by using the fact that the numerator is a polynomial function evaluated at x and polynomials are continuous at every real number (in particular they are continuous at π). So the conditions of Case 2 have been satisfied, and thus $\displaystyle\lim_{x\to\pi^+}\frac{x^2-3x+2}{\sin(x)}=-\infty$. In words we say $\dfrac{x^2-3x+2}{\sin(x)}$ decreases without bound as x approaches π from the right.

3. $\displaystyle\lim_{t\to 2^-}\frac{t^2-t-2}{t^2-t-6}$

$\displaystyle\lim_{t\to 2^-}(t^2-t-2)=2^2-2-2=0$. Also, $\displaystyle\lim_{t\to 2^-}(t^2-t-6)=2^2-2-6=-4$. Therefore, the conditions of Limit Law 5 on page 35 have been verified. Thus, we apply this limit law and get

$$\lim_{t\to 2^-}\frac{t^2-t-2}{t^2-t-6}=\frac{\lim_{t\to 2^-}(t^2-t-2)}{\lim_{t\to 2^-}(t^2-t-6)}=\frac{0}{-4}=0$$

4. $\lim\limits_{x\to 2^-} \dfrac{y^2-9}{|y-2|}$

First of all, this limit was supposed to be $\lim\limits_{y\to 2^-} \dfrac{y^2-9}{|y-2|}$ so that is the one I will solve. For $y < 2$, $|y-2| > 0$. Also $\lim\limits_{y\to 2^-} |y-2| = 0$. Also, $\lim\limits_{y\to 2^-} (y^2-9) = 2^2 - 9 = -5$. By Case 3 it follows that $\dfrac{y^2-9}{|y-2|}$ decreases without bound as y approaches 2 from the left, or $\lim\limits_{y\to 2^-} \dfrac{y^2-9}{|y-2|} = -\infty$.

5. $\lim\limits_{x\to(-4)^-} \dfrac{x^2+3x}{x^2+x-12}$

$\lim\limits_{x\to(-4)^-} (x^2+3x) = (-4)^2 + 3(-4) = 4 > 0$, and $\lim\limits_{x\to(-4)^-} (x^2+x-12) = 0$. Factoring the denominator we get $x^2+x-12 = (x+4)(x-3)$, and for $x < -4$ it follows that $x+4 < 0$ while $x-3 < -7 < 0$. Since the product of two negative numbers is positive, we see that $x^2+x-12 > 0$ for $x < -4$. The conditions of Case 1 have been shown to hold, and thus
$\lim\limits_{x\to(-4)^-} \dfrac{x^2+3x}{x^2+x-12} = \infty$. The proper way to say this in words is $\dfrac{x^2+3x}{x^2+x-12}$ increases without bound as x approaches -4 from the left.