MTH-145-04 **Typical Problems for Section 2.5**

1. A manufacturer's total cost is $C(x) = 10\sqrt{x} - 0.2x + 300$ thousands of dollars, where x is the number of units produced.

a. Find the total cost of producing 100 units. $C(100) = 10\sqrt{100} - 0.2(100) + 300$

$= 100 - 20 + 300 = 380$

To produce 100 units costs \$380,000.

b. Write down the expression that represents the cost of producing the 101st unit.

$\big(C(101) - C(100)\big) \cdot 1000$ is the exact cost in dollars to produce the 101st unit.

c. Use marginal analysis to estimate the cost of producing the 101st unit.

$$C'(x) = \frac{d}{dx}\left(10x^{1/2} - 0.2x + 300\right) = 5x^{-1/2} - 0.2 = \frac{5}{\sqrt{x}} - 0.2$$

$$C(101) - C(100) \approx C'(100) \cdot 1 = \frac{5}{\sqrt{100}} - 0.2 = \frac{5}{10} - 0.2 = 0.3$$

Marginal analysis estimates the cost of producing the 101st unit to be $(0.3) \cdot 1000 = \$300$.

2. Estimate how much the function $f(x) = \dfrac{x-2}{x+5} + 3$ will change as x changes from 5 to 4.5.

$$f'(x) = \frac{(x+5)\cdot 1 - (x-2)\cdot 1}{(x+5)^2} = \frac{7}{(x+5)^2}$$ Let $h = -0.5$

$$f(5+h) - f(5) = f(4.5) - f(5) \approx f'(5)(-0.5)$$

$$= \frac{7}{(5+5)^2}(-0.5) = \frac{7}{25}(-.5)$$

So, the change in $f(x)$ as x changes from 5 to 4.5 is approximately $-\dfrac{3.5}{25}$.

$$= -\frac{3.5}{25}$$