Pigeonhole Examples

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Mathematics 110
Spring 2017

Statement of PHP

Pigeonhole Principle

Suppose that n and m are positive integers with $\mathbf{m > n}$. Regardless of how we distribute m objects into n boxes, there will always be a box that contains **at least 2** of the objects.

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Generalized Pigeonhole Principle

Suppose that n and m are positive integers with $\mathbf{m > n}$. Regardless of how we distribute m objects into n boxes, there will always be a box that contains **at least** m/n of the objects.

Examples

Prove that if **any** set S of 21 numbers is chosen from $\{1, 2, 3, \ldots, 40\}$ there will always be two numbers in S whose sum is 41.

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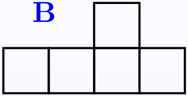
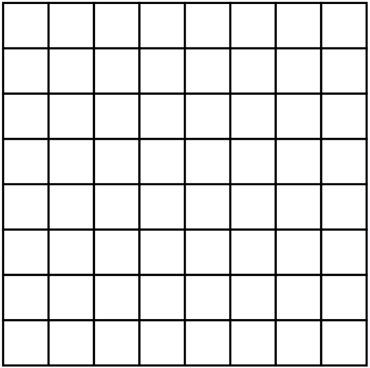
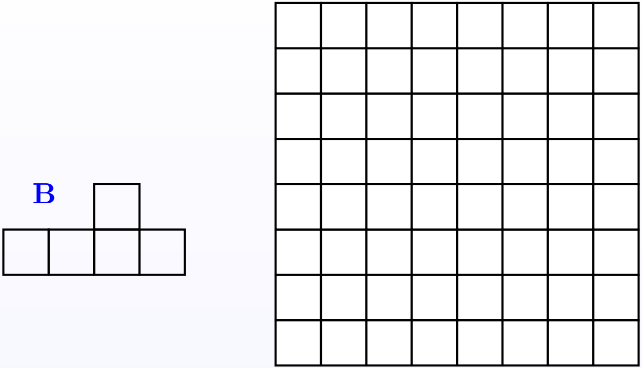
$78,450$ fans attended a Clemson football game one Saturday. The ages of the fans ranged from 6 to 88 inclusive, and their weights (to the nearest pound) ranged from 48 to 315 pounds. Prove there were at least $\mathbf{4}$ fans in attendance who were the exact same age and had the exact same weight.

B







- Imagine all the ways that the puzzle piece **B** could be placed on this chess board having the same orientation as shown.

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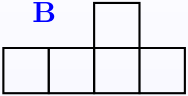
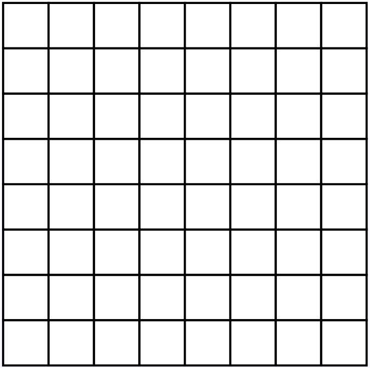
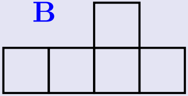
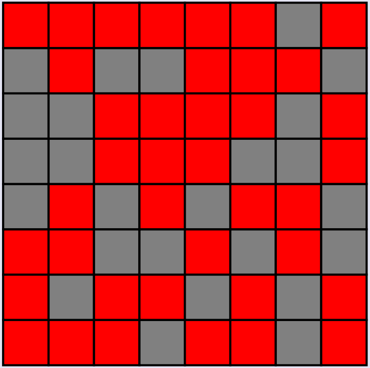




- Imagine all the ways that the puzzle piece **B** could be placed on this chess board having the same orientation as shown.
- Imagine all the different patterns that are possible if we color each of the five squares in **B** either red or gray.

B





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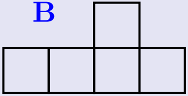
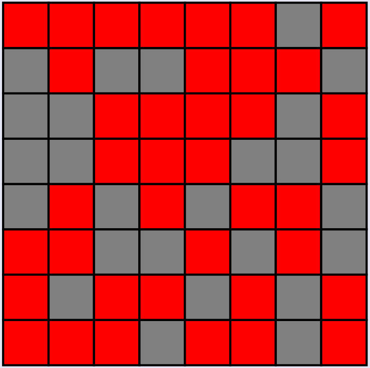
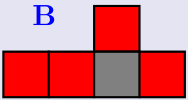
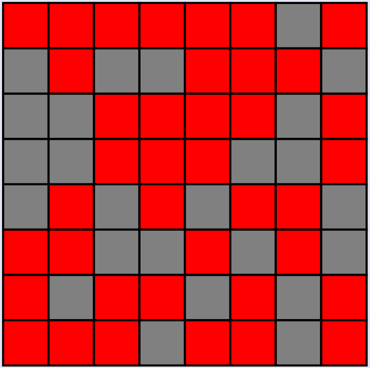




Prove that regardless of how the 64 squares of the 8×8 chess board are colored with red and gray there will always be (at least) two copies of **B**, with the given orientation, somewhere on this colored board that have the same color pattern.

A Particular Pattern That Occurs More Than Once





Three occurrences of the indicated pattern. One marked by "x", another by "y" and a third by "z".

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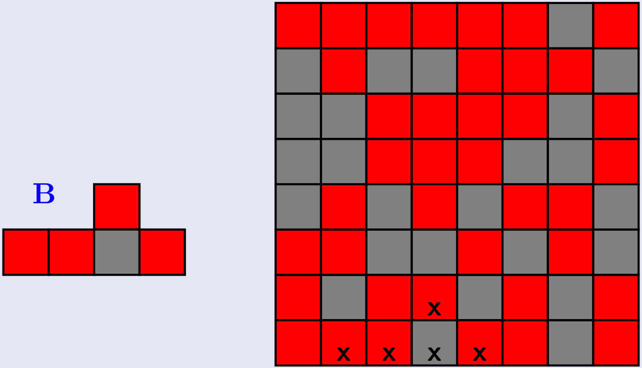

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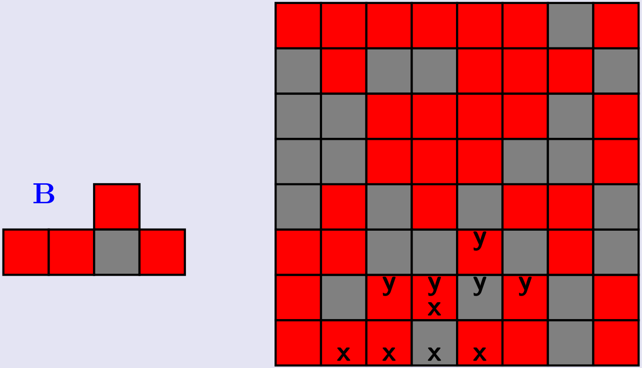

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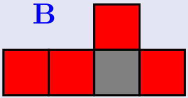

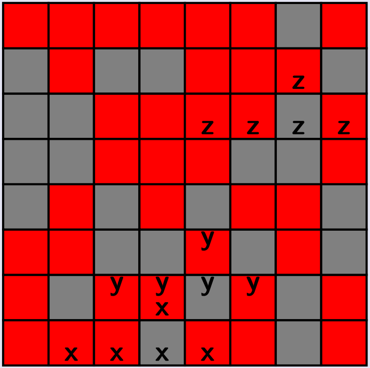

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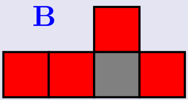

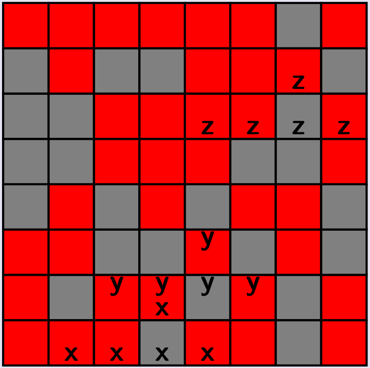

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