Chapter 8, Section 1

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Outline

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- Definition of Markov Chain
- Properties and Examples
- Transition Diagrams and Transition Matrices

Markov Chain

Definition

A **Markov chain** or **Markov process** is a multistage experiment such that:

- At the end of each stage the process is one of a fixed finite number of "states."
- For any two of these states, say A and B, the conditional probability of a transition from A to B depends only on A and B. In particular, this conditional probability does not depend on how the process got to state A.

Markov Chain

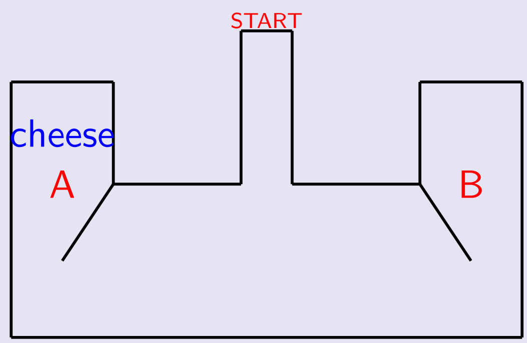
Example

Commuters to Philadelphia can be divided into two groups: those who commute by car (C) and those who commute by public transportation (P). Suppose that each year 5% of the car commuters switch to public transportation the following year, and 10% of the commuters using public transportation switch to commuting by car the following year.

Draw a tree diagram and a transition diagram to represent this Markov chain.

In an attempt to see how mice can learn, 100 mice are run repeatedly through a maze. A mouse is placed at the start and removed from the maze after it enters either Room A or Room B. A mouse entering A is rewarded with cheese, while a mouse entering B receives no reward. When the experiment is performed many times it is discovered that 80% of the mice that ended in Room A the last time end in Room A again, and the other 20% end in Room B. Of the mice that ended in Room B last time, 60% end in Room A the next time and 40% end in Room B the next time.

Example



Example

Two spots on the floor are marked – one is red and the other is green. Near the red spot is a box that contains one red poker chip and one green poker chip. Near the green spot is a box containing two red poker chips and one green poker chip. A person starts on the red spot, selects a chip at random from the nearby box, replaces the chip and moves to the spot having the same color as the chip that was selected. This process continues, each time the chip is selected from the box near the spot on the floor.

Use tree diagrams and transition diagrams to find

1. the probability the person is on the green spot after 1 transition.
2. the probability the person is on the green spot after 2 transitions.
3. the probability the person is on the red spot after 3 transitions.
4. the probability the person is on the green spot after 20 transitions.

Markov Chain

When a Markov chain is used to model the transitions of people, animals or objects from one "state" to another, we can then approximate how an initial distribution of a number of these people, animals or objects changes over time.

The tax accounting firm *Taxes-R-Us* allows its tax accountants to either work in the office or work online from home. However, each accountant must work in a given place for a month at a time. At the end of each month an accountant can choose whether to change his/her working location. The firm knows from experience that an accountant who is working in the office in a given month will choose to work in the office the following month with probability 0.8, and one who is working from home in a given month will choose to work from home the following month with probability 0.6.

Markov Chain

The rental car company **Rentme.com** has rental offices in three cities A, B and C. **Rentme.com** rents cars for a week at a time. From its rental history the company knows that a car rented in city A will be returned to the office in A, B or C with probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$, respectively. A car rented in city B will be returned to the office in A, B or C with probabilities $\frac{1}{3}, \frac{1}{3}$ and $\frac{1}{3}$, respectively. Finally, a car rented from **Rentme.com** in city C will be returned to the office in A, B or C with probabilities $\frac{3}{4}, 0$ and $\frac{1}{4}$, respectively. We will assume that **Rentme.com**'s business is so good that every car it owns is rented each week.