Chapter 7, Section 3

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Fall 2014

Solutions of Linear Programming Problems

Outline

- Bounded and unbounded sets
- Corner points
- Solution method

Bounded or Unbounded?

A set of points in the xy-plane is **bounded** if it is contained inside some circle that is centered at $(0, 0)$. If this is not the case the set is called **unbounded**.

A point (a, b) is a **corner point** of a feasible set for a linear programming problem if (a, b) is the intersection of two or more boundary lines of the feasible set.

Example

A

Find the maximum of $5x + 4y$ subject to

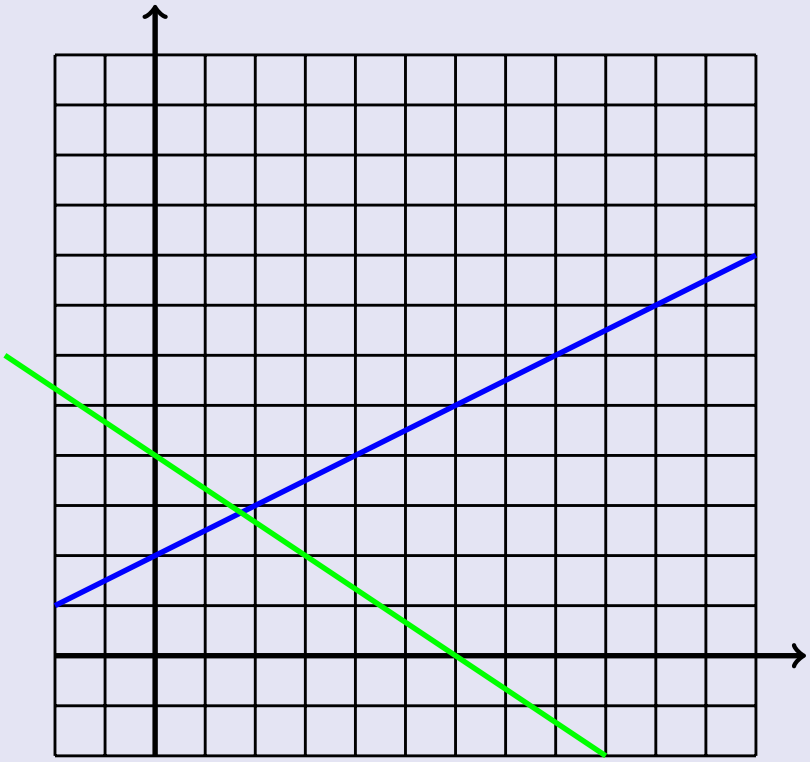
$$\begin{aligned} 2x + 3y &\leq 12 \\ -x + 2y &\leq 4 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

B

Find the maximum of $5x + 4y$ subject to

$$\begin{aligned} -x + 3y &\geq -3 \\ -x + 2y &\leq 4 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

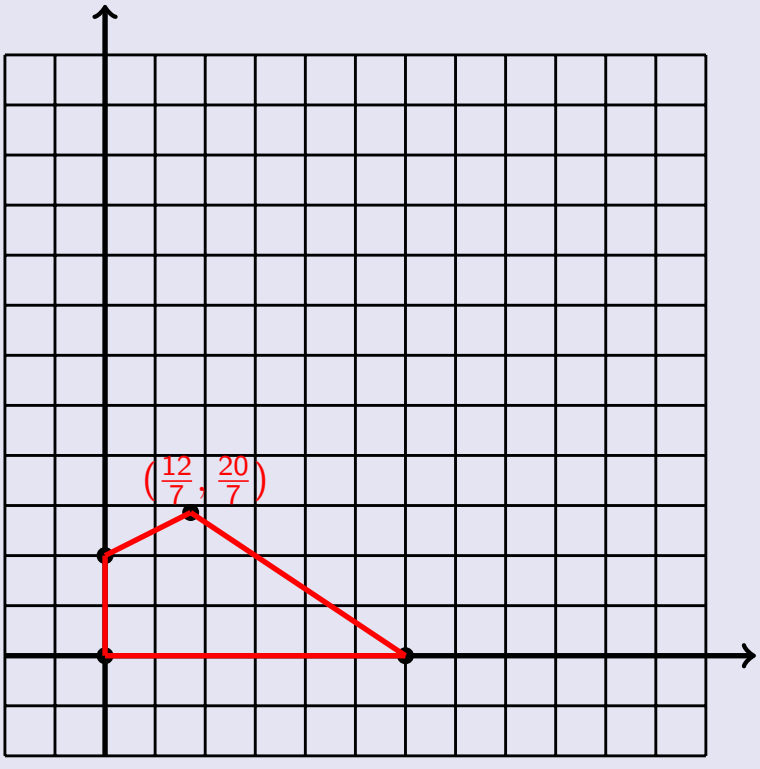
A



maximize
$5x + 4y$
where

$2x + 3y \leq 12$
$-x + 2y \leq 4$
$x \geq 0$
$y \geq 0$

A

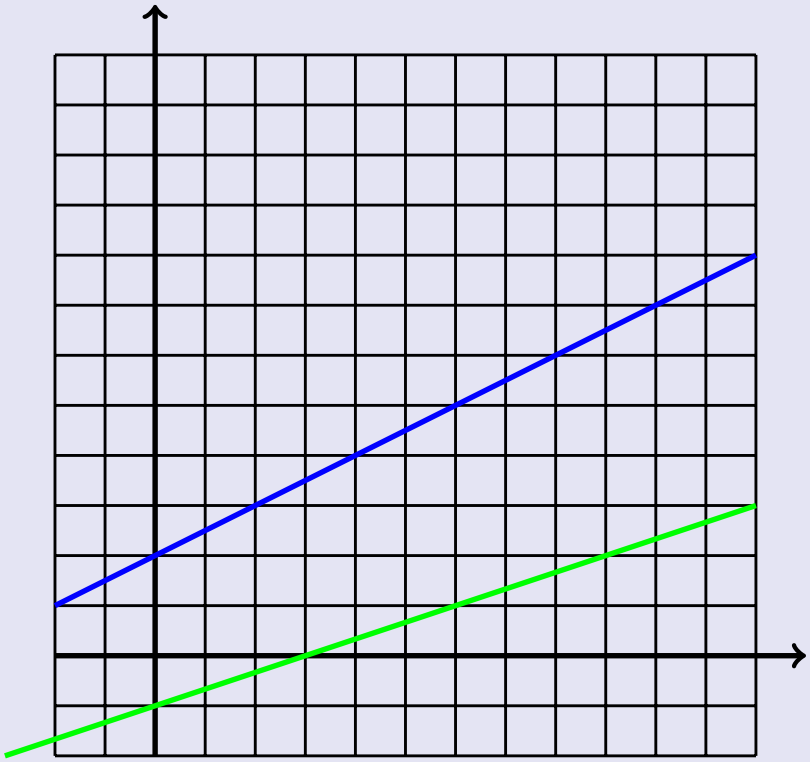


maximize
$5x + 4y$
where

$2x + 3y \leq 12$
$-x + 2y \leq 4$
$x \geq 0$
$y \geq 0$

bounded
feasible set

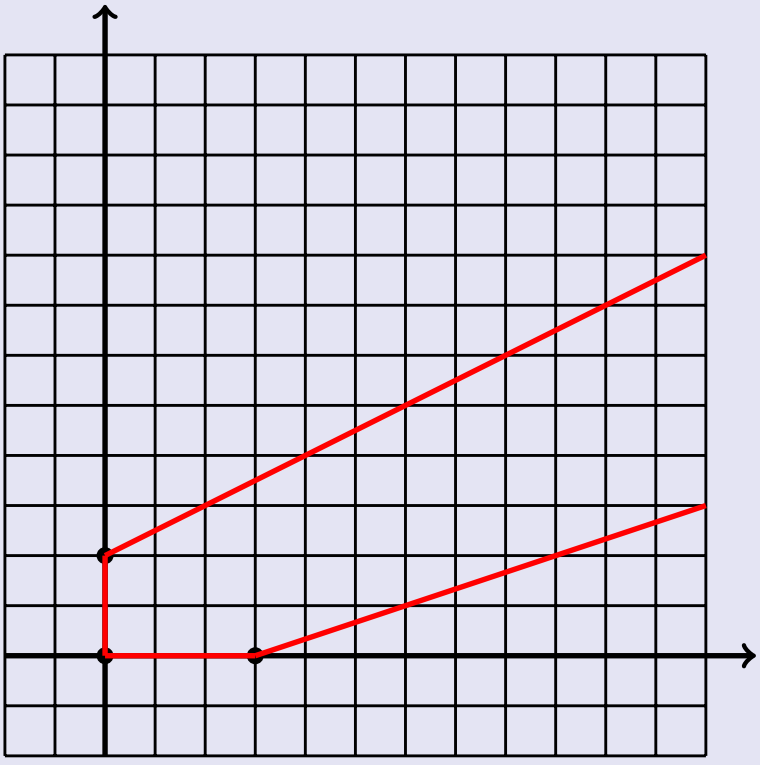
B



maximize
$5x + 4y$
where

$-x + 3y \geq -3$
$-x + 2y \leq 4$
$x \geq 0$
$y \geq 0$

B

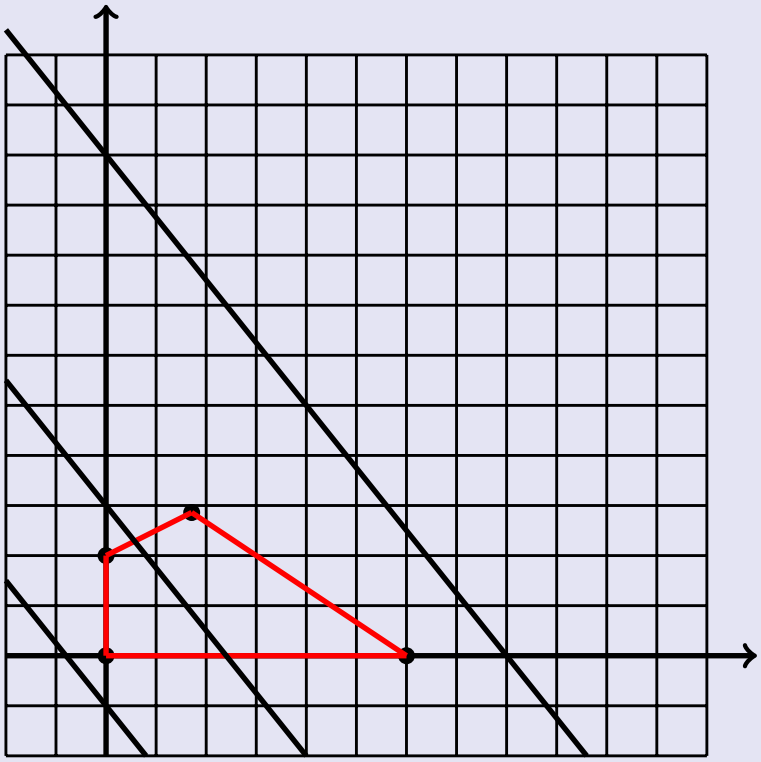


maximize
$5x + 4y$
where

$-x + 3y \geq -3$
$-x + 2y \leq 4$
$x \geq 0$
$y \geq 0$

unbounded
feasible set

A

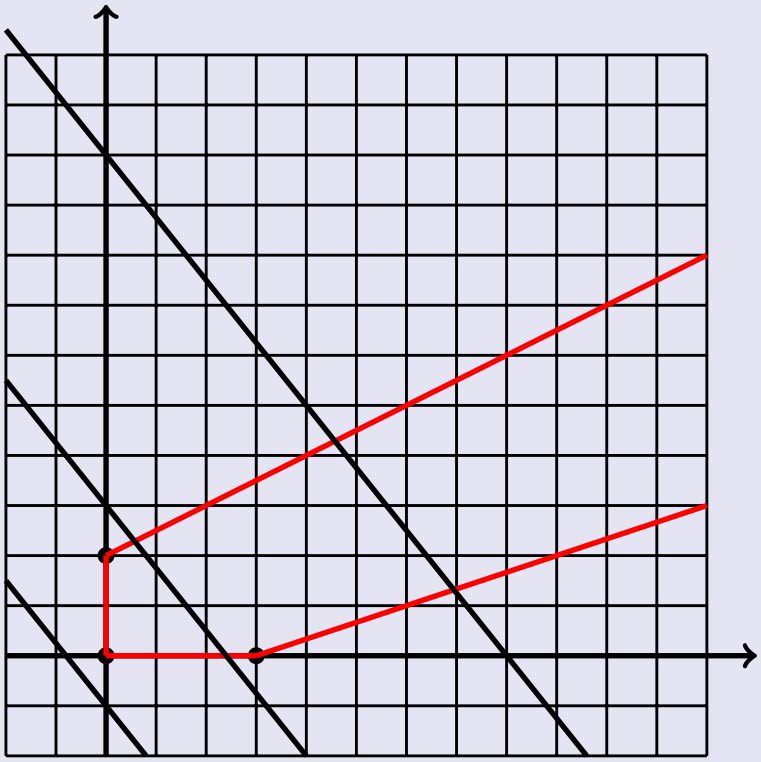


maximize
$5x + 4y$

$5x + 4y = -4$
$5x + 4y = 12$
$5x + 4y = 40$

Which line goes with which equation?

B



maximize
$5x + 4y$

$5x + 4y = -4$
$5x + 4y = 12$
$5x + 4y = 40$

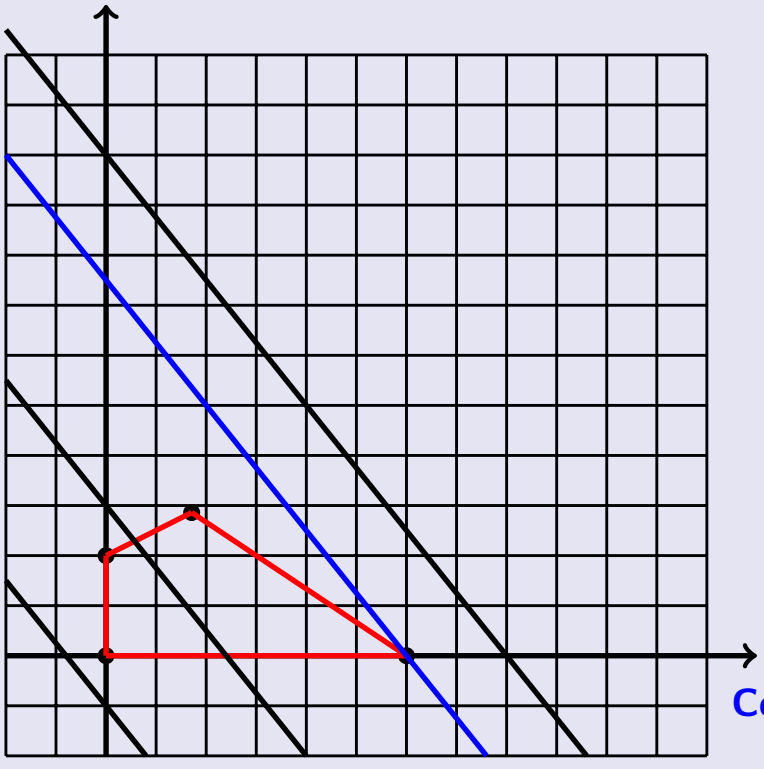
Which line goes with which equation?

Result on page 331: Read it!

Theorem Suppose S is the feasible set for a linear programming problem and let **p** be the objective function.

1. If **S** is bounded, then **p** has both a maximum value and a minimum value. These maximum and minimum values each occur at corner points.
2. If **S** is unbounded and has at least one corner point, then exactly one of the following is true:
   - **p** has a maximum value and it occurs at a corner point.
   - **p** does not have a maximum value on **S**.
3. If **S** is unbounded and has at least one corner point, then exactly one of the following is true:
   - **p** has a minimum value and it occurs at a corner point.
   - **p** does not have a minimum value on **S**.

A



maximize
$\mathbf{5x + 4y}$

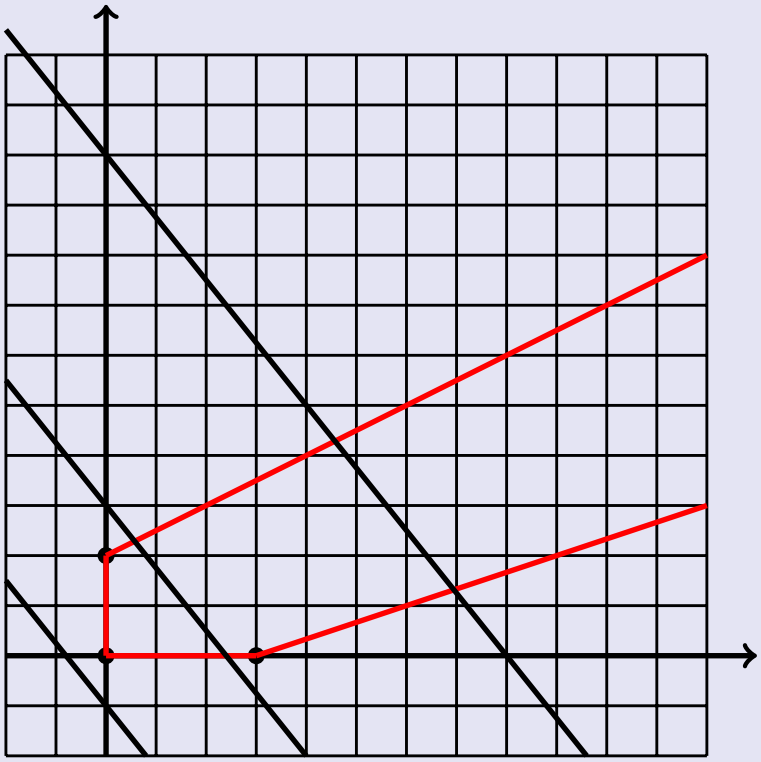
$\mathbf{5x + 4y = -4}$
$\mathbf{5x + 4y = 12}$
$\mathbf{5x + 4y = 40}$

$\mathbf{5x + 4y = 30}$

maximum is 30

Computation table!

B



maximize
$5x + 4y$

$5x + 4y = -4$
$5x + 4y = 12$
$5x + 4y = 40$

no maximum

Solution Method: Computation Table

Method: Suppose S is the feasible set for a linear programming problem and let **p** be the objective function.

1. If **S** is bounded, then **p** has both a maximum value and a minimum value. These maximum and minimum values occur at corner points. Evaluate the objective function at each corner point. Choose the largest value if the objective is to maximize. Choose the smallest value if the objective is to minimize.
2. If **S** is unbounded and has at least one corner point, then evaluate the objective function at each corner point. Follow the procedure on page 335 of textbook.