Chapter 5, Section 2

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Fall 2014

Systems of Linear Equations in Two Variables

Outline

- Translating Data Relationships into Equations
- Solving Systems using Substitution

Translating Example

A truck rental company has two types of trucks to rent. Each Type A truck has 20 cubic feet of refrigerated space and 40 cubic feet of non-refrigerated space, while each truck of Type B has 30 cubic feet of refrigerated space and 30 cubic feet of non-refrigerated space. Foods-R-Us has to ship 900 cubic feet of refrigerated produce and 1200 cubic feet of non-refrigerated produce. How many trucks of each type should Foods-R-Us rent so that they can ship all the produce without wasting any of the space in any rental truck?

Summarize the data in a table.

Let $x =$ the number of Type A trucks that will be rented.
Let $y =$ the number of Type B trucks that will be rented.
Write an equation for refrigerated space.
Write an equation for non-refrigerated space.

Translating Example (Exercise#2)

Suppose that Murphy's Muffin Shoppe decides to make both large and small bran muffins. Each large muffin uses 4 ounces of dough and 2 ounces of bran, while a small muffin uses 1 ounce of dough and 1 ounce of bran. Suppose also that there are 300 ounces of dough available each day and 160 ounces of bran. Formulate a system of equations to determine how many muffins of each size should be baked each day to use up all the dough and all the bran.

Summarize the data in a table.

Let $x =$ the number of large muffins to bake.
Let $y =$ the number of small muffins to bake.
Write an equation for using all the dough.
Write an equation for using all the bran.

Solving Systems of Linear Equations

Terminology

If a system of two linear equations has a solution (that is, if the lines they represent have a point of intersection), we call the system **consistent**. If the system does not have a solution (that is, if the lines they represent are parallel (have the same slope)), we call the system **inconsistent**.

Solving Systems of Linear Equations

Solving by Substitution

Solve one of the equations for one variable to get an expression in terms of the other variable. Substitute this expression into the other equation resulting in a linear equation involving only one variable. Solve for this variable and then work backwards to solve for the other variable.

Example

Solve

$$\begin{aligned} 2x + y &= 12 \\ 3x - 5y &= 5 \end{aligned}$$

Exercise #8

Solve

$$\begin{aligned} 3x - 2y &= 3 \\ 3x - 8y &= -6 \end{aligned}$$

Example

Solve

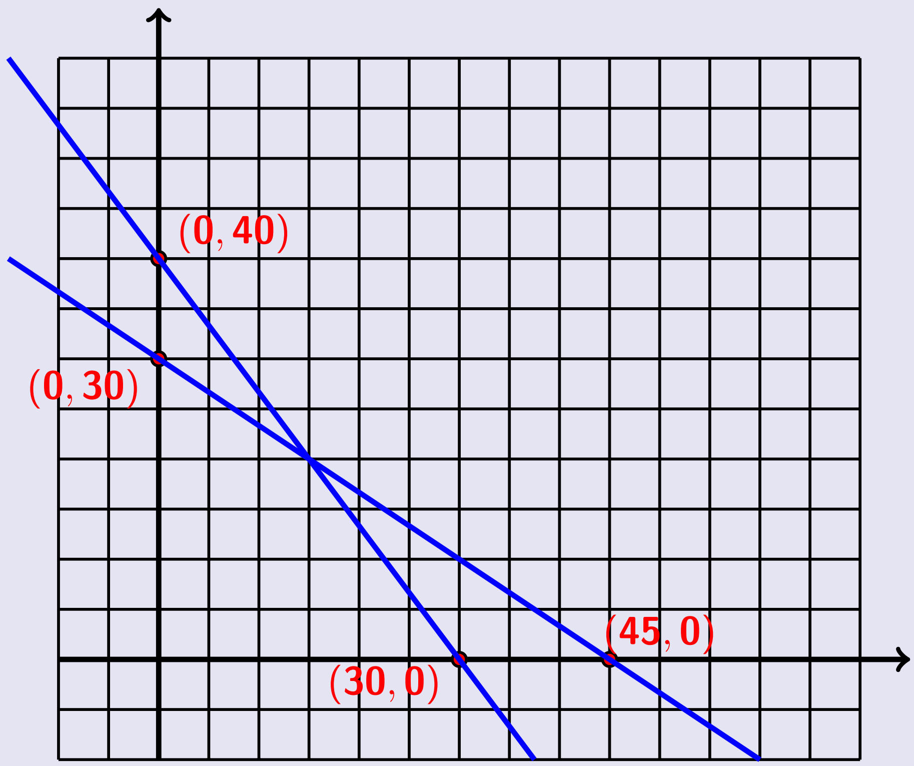
$$\begin{aligned} x - 5y &= 1 \\ -4x + 20y &= 5 \end{aligned}$$

Example

A truck rental company has two types of trucks to rent. Each Type A truck has 20 cubic feet of refrigerated space and 40 cubic feet of non-refrigerated space, while each truck of Type B has 30 cubic feet of refrigerated space and 30 cubic feet of non-refrigerated space. Foods-R-Us has to ship 900 cubic feet of refrigerated produce and 1200 cubic feet of non-refrigerated produce. How many trucks of each type should Foods-R-Us rent so that they can ship all the produce without wasting any of the space in any rental truck?

Let $x =$ the number of Type A trucks that will be rented.
Let $y =$ the number of Type B trucks that will be rented.

$$\begin{aligned} 20x + 30y &= 900 \\ 40x + 30y &= 1200 \end{aligned}$$



(Exercise#2)

Suppose that Murphy's Muffin Shoppe decides to make both large and small bran muffins. Each large muffin uses 4 ounces of dough and 2 ounces of bran, while a small muffin uses 1 ounce of dough and 1 ounce of bran. Suppose also that there are 300 ounces of dough available each day and 160 ounces of bran. Formulate and solve a system of equations to determine how many muffins of each size should be baked each day to use up all the dough and all the bran.

Let $x =$ the number of large muffins to bake.
Let $y =$ the number of small muffins to bake.

$$\begin{aligned} 4x + y &= 300 \\ 2x + y &= 160 \end{aligned}$$

(Exercise#18)

A lottery winner plans to invest part of her $1,000,000 in utility bonds paying 12 percent per year and the rest in a savings account paying 8 percent per year. How much should be allocated to each investment if the income from the savings account is to be twice the income from the utility bonds?

Let $x =$ the amount to invest in utility bonds
Let $y =$ the amount to invest in the savings account

$$\begin{aligned} x + y &= 1,000,000 \\ .08y &= 2(.12x) \end{aligned}$$

(Exercise#28)

Murphy's Muffin Shoppe makes large and small apple-raisin muffins. A large apple-raisin muffin requires 5 ounces of dough, 2 ounces of apples, and 0.5 ounce of raisins. a small apple-raisin muffin requires 3 ounces of dough, 1 ounce of apples, and 0.3 ounce of raisins. Murphy has 270 ounces of dough, 100 ounces of apples, and 30 ounces of raisins. How many muffins of each type can Murphy make to use all the dough and apples? If he does so, how many ounces of raisins remain unused?

Let $x =$ the number of large muffins to make
Let $y =$ the number of small muffins to make

$$\begin{aligned} 5x + 3y &= 270 \\ 2x + y &= 100 \end{aligned}$$