

CHAPTER 5, SECTION 1

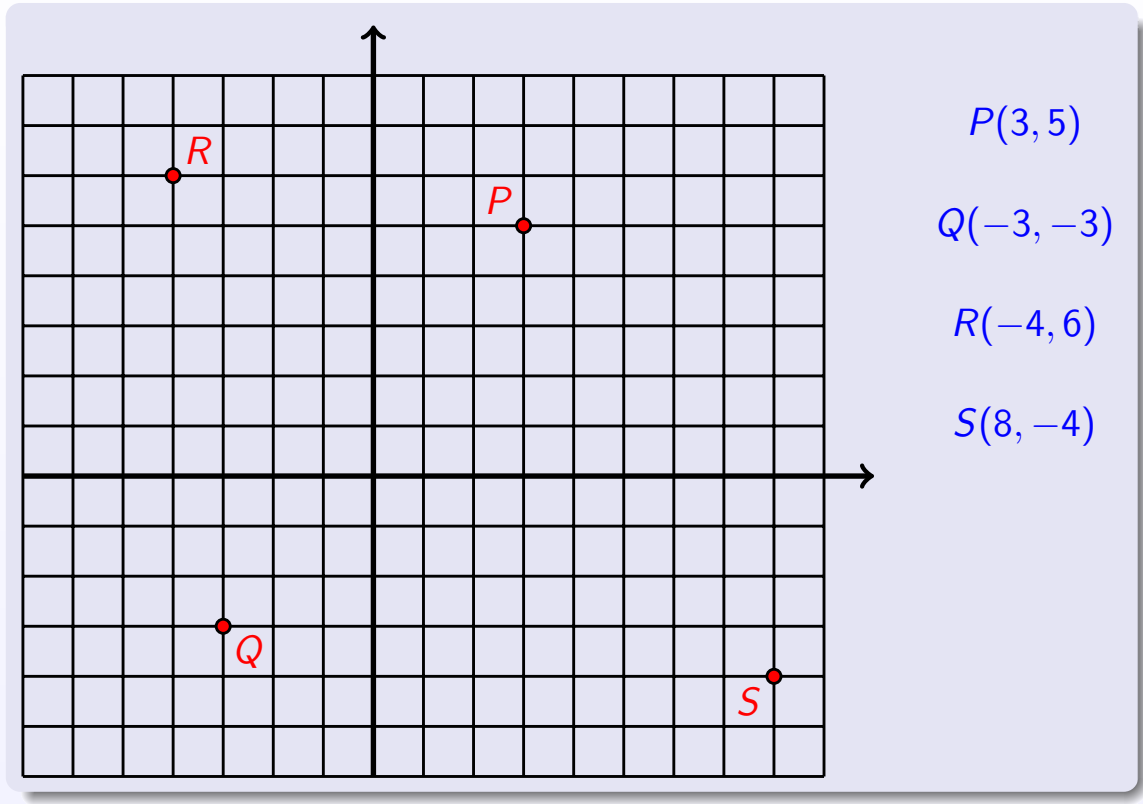
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Outline

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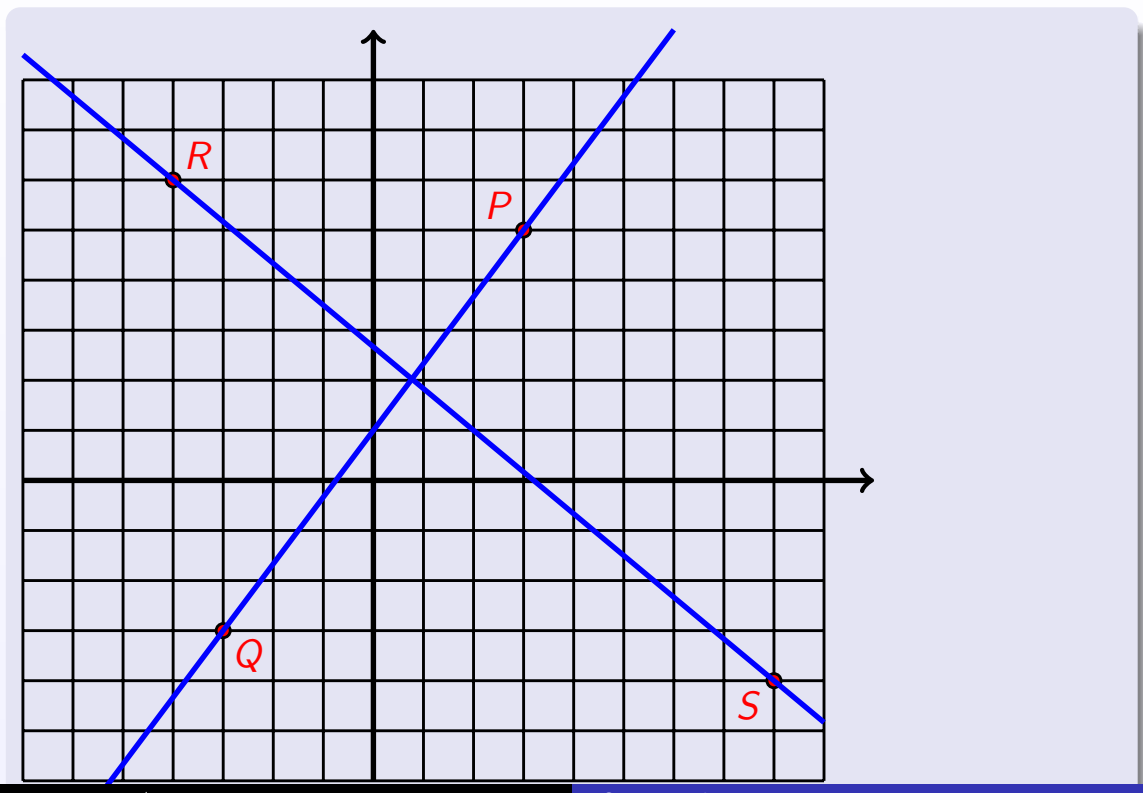
- Cartesian Coordinate System
- Lines and Their Equations
- Slopes and Intercepts

Cartesian Coordinates



Lines

Any two points determine a unique (straight) line.



Equation of Line

Let A , B and C be fixed numbers such that at least one of A or B is not zero. The set of all points (x, y) such that $Ax + By = C$ is a line. A point (u, v) is on this line if $Au + Bv = C$. A point (d, e) is not on this line if $Ad + Be \neq C$. $Ax + By = C$ is called a **linear equation**.

$P(3, 5)$ $Q(-3, -3)$ $R(-4, 6)$ and $S(8, -4)$

The line determined by P and Q on previous slide has equation $-4x + 3y = 3$.

The line determined by R and S on previous slide has equation $5x + 6y = 16$.

Equation of Line

Intercepts

The **x-intercept** of a line is the point where it crosses or intersects the x-axis. The **y-intercept** of a line is the point where it crosses or intersects the y-axis.

A line $Ax + By = C$ has an x-intercept if $A \neq 0$; the x-intercept is then $(\frac{C}{A}, 0)$.

A line $Ax + By = C$ has an y-intercept if $B \neq 0$; the y-intercept is then $(0, \frac{C}{B})$.

Examples

Find the x-intercept and y-intercept of each of the following and then draw their graph:

(i) $3x - 5y = 20$

(ii) $4y = 19$

Slope of a Line

The **slope** of a non-vertical line is the ratio of the difference in y coordinates to the difference in x coordinates using any two different points on the line. If (x_1, y_1) and (x_2, y_2) are two different points on a line and $x_1 \neq x_2$, the slope of the line is the number m defined by

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}.$$

Examples

Find the slope of each of the following lines and then graph them:

- (i) $3x - 5y = 20$
- (ii) $4y = 19$
- (iii) $4x + y = 12$

A line has equation $Ax + By = C$ and $B \neq 0$. The equation of the line can be written in the equivalent form $y = -\frac{A}{B}x + \frac{C}{B}$ (called the **slope-intercept** form). How? What is the slope of this line?

Exercise #14

Find the equation of each of the following lines:

- (a) The line has slope -3 and it contains the point $(1, 0)$.
- (b) The line has slope 0 and it contains the point $(1, -2)$.
- (c) The slope of the line is undefined and it contains the point $(3, 2)$.
- (d) The line contains the point $(4, -7)$ and $(6, -3)$.

Exercise #12

Suppose that the cost of a truck rental is related to the number of days the truck is rented by a linear equation. The cost of a 2-day rental is \$105, and the cost of a 6-day rental is \$285. Find the cost of a 7-day rental.

Exercise #24

Suppose that the cost of leasing temporary office space is related to the length of the lease by a linear equation. Also, suppose that the cost of a 60-day lease is \$5000, and the cost of a 90-day lease is \$7250. If a start-up corporation has \$11,000 to use for office space, what length lease can it obtain?