

CHAPTER 4, SECTION 2

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Outline

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- Definition of Expected Value
- Expected Value of Binomial Random Variables
- Examples

Intuitive Idea

A box contains 4 cards: the 2, 3 and 5 of spades and the 5 of clubs. You get to select a card at random from the box and I pay you the number of dollars that is equal to the rank of the card. To make this a fair game (that is, so that neither you nor I have an advantage), how much should you pay me to play the game? Consider what you expect to happen if the game is played 100 times.

Definition

Suppose X is a random variable with range $\{x_1, x_2, \dots, x_m\}$ and density function given in the table

Value k of X	Probability $\Pr[X = k]$
x_1	$p_1 = \Pr[X = x_1]$
x_2	$p_2 = \Pr[X = x_2]$
\cdot	\cdot
\cdot	\cdot
x_m	$p_m = \Pr[X = x_m]$

The **expected value** (or **mean**) of X is the number

$$E(X) = x_1 p_1 + x_2 p_2 + \cdots + x_m p_m .$$

Examples

Example

A fair coin is tossed until you get a head or until the coin has been tossed six times. Let N be the random variable that denotes the number of times the coin is tossed. Find the density function and the expected value of N .

Example

A box contains 3 red, 2 blue and 1 green poker chips. A set of 2 chips is selected simultaneously and at random. Let C denote the number of red chips selected. Find $E(C)$.

Examples

Example

An electronics store Worst Cell offers a one-year warranty on the smart phones they sell. A warranty costs the customer \$35, and it costs Worst Cell \$250 to replace a smart phone that goes bad. Worst Cell knows that 2% of the smart phones they sell will need to be replaced during their first year of use. Let W be the random variable that denotes the profit for Worst Cell on each phone they sell. Find the expected profit for Worst Cell on each one-year warranty it sells.

Example

The *Super-Duper Drawing* has 1 Grand Prize of \$100,000, 2 Big Prizes of \$10,000 each and 7 OK Prizes of \$100 each. Tickets for the drawing cost \$2. Suppose that 100,000 tickets were sold. Find the expected value of a purchased ticket to the Super-Duper Drawing.

Binomial Random Variable

Recall that a binomial random variable B counts the number of successes in a Bernoulli process.

Suppose a Bernoulli process of n trials has $p = \Pr[\text{success}]$. Then

$$E(B) = np.$$

Example

The verbal part of the GRE consists of 100 multiple choice questions. There are 4 possible choices of answers for each question. Johnny takes this test and randomly guesses an answer for each question. What is the expected number of questions that Johnny will answer correctly?

score = # correct - 1/4 point for each question answered incorrectly