Chapter 4, Section 1

Doug Rall
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Outline

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- Definition of Random Variable
- Binomial Random Variable
- Probability Density Functions

Random Variables

Example

A box contains 3 blue tiles, 2 red tiles and 3 yellow tiles. You pay me \$1 to play the following game. You select 2 tiles from the box, simultaneously and at random. If the tiles are the same color, I pay you \$3. If the tiles are of different colors, you win nothing. Will you play? Once? Many times? How do you decide?

Definition

A **random variable** X on a sample space S is an assignment of numbers to the elements of S, with exactly 1 number assigned to each outcome. The **range of** X is the set of all possible values of X.

Binomial Random Variables

Definition

A **binomial random variable** is a random variable that assigns to each outcome of a Bernoulli process the number of successes.

Example

A test consists of six multiple choice questions; each question has four possible answers, exactly one of which is correct. A student takes this test by randomly guessing an answer for each question. Let C be the random variable that records the number of questions answered correctly. Find the possible values of C (i.e., the range of C) and for each number in this range, find the probability that C takes on this value. C is a binomial random variable.

Density Function

Definition

Suppose that X is a random variable defined on a sample space S and let R be the range of X. The function f defined for each $a \in R$ by

$$\mathbf{f(a) = \Pr[X = a]}$$

is called the **density function** of X.

Example

Roll a pair of fair dice. Let T be the random variable that computes the sum of the numbers that appear up on the dice. Find the density function of T.

Examples

Example

A fair coin is tossed until you get a head or until the coin has been tossed six times. Let N be the random variable that denotes the number of times the coin is tossed. Find the range of N and the density function of N.

Example

A 3-card hand is dealt at random from a standard deck of cards. Let D denote the number of diamonds in the hand. Find the range and the density function of D.

Example

A box contains four poker chips labeled 1,2,3 and 4. Four poker chips are selected at random from the box, one after the other without replacement. The random variable B is the sum of the labels on the poker chips selected before poker chip 4 is selected. Find the range and the density function of B.

Example

An electronics store Worst Cell offers a one-year warranty on the smart phones they sell. A warranty costs the customer \$35, and it costs Worst Cell \$250 to replace a smart phone that goes bad. Worst Cell knows that 2% of the smart phones they sell will need to be replaced during their first year of use. Let W be the random variable that denotes the profit for Worst Cell on each phone they sell. Find the range and the density function of W.