

CHAPTER 3, SECTION 5

Doug Rall
Fall 2014

Outline

Outline

- Definition of Bernoulli Process
- Bernoulli Process Probability

Bernoulli Trials

Definition: A **Bernoulli trial** is an experiment with exactly two possible outcomes. These two outcomes are typically called **success (S)** and **failure (F)**.

Examples

- A fair coin is tossed resulting in a Head(S) or Tail(F).
- A multiple choice question (with five possible choices) is answered by random guessing, resulting in a correct(S) or an incorrect(F) answer.
- A place kicker attempts extra points resulting in one point(S) for the team or in zero points(F).

Bernoulli Trials

Definition: In a Bernoulli trial we let $p = \Pr[S]$ and $q = \Pr[F]$. Note that since the only two possible outcomes are S and F , we get

$$p + q = 1.$$

Examples

- A fair coin is tossed resulting in a Head(S) or Tail(F).
 $p = \Pr[S] = 1/2$ and $q = \Pr[F] = 1/2$.
- A multiple choice question (with five possible choices) on a test is answered by random guessing resulting in a correct(S) or an incorrect(F) answer. $p = \Pr[S] = 1/5$, $q = \Pr[F] = 4/5$
- A place kicker attempts extra points resulting in one point(S) for the team or in zero points(F). Suppose he has made 95% of extra point attempts in his career so far.
 $p = \Pr[S] = .95$ and $q = \Pr[F] = .05$

Bernoulli Process

Definition: A **Bernoulli process** is a sequence of repetitions of the same Bernoulli trial. The repetitions are independent, and so the probability of success (and of failure) remains the same on each trial.

Examples

- A fair coin is tossed 5 times resulting in a Head(S) or Tail(F) on each toss. On each toss, $p = \Pr[S] = 1/2$ and $q = \Pr[F] = 1/2$.
- A multiple choice test with 10 questions (with five possible choices on each question) is completed by randomly guessing the answer to each question. On each question, $p = \Pr[S] = 1/5$ and $q = \Pr[F] = 4/5$.
- A place kicker attempts extra points resulting in one point(S) for the team or in zero points(F). During a game he attempts five extra points.

Bernoulli Process

For a given Bernoulli process that has n trials we would like to be able to answer the following types of questions:

- What is the probability of exactly 3 successes?
- What is the probability of at most 2 failures?
- What is the probability of at least 4 successes?
- What is the probability of no successes?

Example

A box contains 2 red balls and 3 green balls. A trial consists of selecting a ball at random from the box, noting its color, and returning the ball to the box. We will consider getting a red ball a success. Our experiment consists of 4 (independent) trials. Draw and label the probability tree for this experiment and then compute

- the probability of selecting exactly 1 red ball;
- the probability of selecting exactly 2 red balls;
- the probability of selecting no red balls.

See page 129 of text!

Main Result

Theorem: If a Bernoulli process with n trials has

$$p = \Pr[S] \quad \text{and} \quad q = \Pr[F]$$

then

$$\Pr[\text{exactly } k \text{ successes in } n \text{ trials}] = C(n, k)p^k q^{n-k}.$$

Example

A certain Bernoulli process has 7 trials with $p = \Pr[S] = 3/4$ and $q = \Pr[F] = 1/4$. Find

- the probability of exactly 6 successes.
- the probability of exactly 4 failures.
- the probability of at least 5 successes.
- the probability of at most 3 successes.

Example

In a 10-question multiple choice test, in which each question has 4 possible choices, find the probability of answering exactly 8 questions correctly just by guessing.

Find the probability of answering at most 3 questions correctly just by guessing.

Exercise #8

A basketball player makes each of his shots with probability 0.5. If he takes 6 shots and if the results of his shots are independent, find the probability that he makes at least 4.

Example

A small car rental agency in Utah rents regular sedans and 4-wheel drive vehicles. Data from the past suggests that about 40 percent of those renting a vehicle want one with 4-wheel drive. On this particular day the agency has lots of sedans but has only two 4-wheel drive vehicles in their current inventory. A tour group of 10 couples is arriving, and each couple will rent a vehicle. Assume that the preferences of this tour group can be viewed as the result of a Bernoulli process and find the probability that every couple that wants a 4-wheel drive can be accommodated.