Chapter 3, Section 4

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Outline

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- Probability Trees
- Bayes Probabilities

Probability Tree

Review: For events A and B in a sample space,

$$\Pr[A \mid B] = \frac{\Pr[A \cap B]}{\Pr[B]}.$$

Rewriting we get **Multiplication Rule for Probability**

$$\Pr[B] \cdot \Pr[A \mid B] = \Pr[A \cap B].$$

Probability Tree

Example

A box labeled B_1 contains two red and one green poker chips. A box labeled B_2 contains one red and three green poker chips. An experiment consists of selecting a box at random and then selecting a poker chip at random from that box.
Draw a probability tree for this experiment.

Find the probability that the chip selected is green.

Find the conditional probability that the chip was selected from B_2 given that the chip is green.

Bayes' Formula

Theorem Let S be the sample space of an experiment and that E_1, E_2 is a partition of S. Let A be any event such that $\Pr[A] > 0$. Then

$$\Pr[E_1 \mid A] = \frac{\Pr[A \mid E_1] \cdot \Pr[E_1]}{\Pr[A \mid E_1] \cdot \Pr[E_1] + \Pr[A \mid E_2] \cdot \Pr[E_2]}$$

DO NOT MEMORIZE!!!

Use Probability Trees

Exercise #4

An experiment has the probability tree shown in Figure 3.17 on page 123 of the text. Find

- $\Pr[B \mid Y]$
- $\Pr[Y \mid B]$
- $\Pr[(X \cup Y) \mid B]$

Exercise #8

There are 3 coins, 2 are fair and 1 is unfair with $\Pr[H] = .6$. A coin is selected at random, flipped twice, and the result of each flip is noted. If the results of the flips are a head followed by a tail, find the probability that a fair coin was selected.

Use Probability Trees

Exercise #18

Students at GSU are being tested for infection with the East Thames virus, and it is estimated that .5 percent of the students are infected. If a student is infected, then the test on that student is positive (that is, the test indicates the student has the virus) 99 percent of the time. If a student is not infected, then the test on that student is negative (that is, the test says the student does not have the virus) 98 percent of the time. If a student is chosen at random from the GSU student population, is tested for the virus and the test results are negative, find the probability that the student actually has the East Thames virus.

Use Probability Trees

Example

Mr. Coffee assembles coffee machines at three manufacturing plants; let's call them A, B and C. Plant A is the largest and produces 40 percent of Mr. Coffee's coffee machines. Plants B and C each produce 30 percent of the total machines made by Mr. Coffee. Historical data shows that 1 percent of the coffee machines produced by plant A are defective, 2 percent of those produced at plant B are defective, and .5 percent of those made at plant C are defective.

A Mr. Coffee machine is selected at random from a store shelf. What is the probability the machine is defective?

A Mr. Coffee machine is selected at random from a store shelf, is tested and is found to be defective. What is the probability that machine was produced at plant B?