Chapter 3, Section 3

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Outline

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- The Multiplication Rule for Probability
- Stochastic Processes
- Probability Trees

Multiplication Rule for Probability

Fact: For events A and B in a sample space,

$$\Pr[A \mid B] = \frac{\Pr[A \cap B]}{\Pr[B]}.$$

Rewriting we get **Multiplication Rule for Probability**

$$\Pr[B] \cdot \Pr[A \mid B] = \Pr[A \cap B].$$

Multiplication Rule: $\Pr[B] \cdot \Pr[A \mid B] = \Pr[A \cap B]$

This is particularly useful when an experiment consists of a sequence of subexperiments in which B is an event that can occur in one stage, A is an event that can occur in a following stage, and we want to know the probability that both A and B occur.

Stochastic Process

A **stochastic process** is an experiment that consists of a sequence of subexperiments.

Probability Trees

A **probability tree** is used to represent and calculate probabilities in a stochastic process.

Exercise #18

Use the properties of a probability tree to find the missing numbers on the probability tree in Figure 3.11.

Probability Trees

Example

A box contains 6 poker chips, 2 red, 3 white and 1 purple. A chip is selected at random, and its color is noted. It is not replaced. A second chip is selected at random, and its color is noted.

Let E = the event that the first chip selected is white.
Let F = the event that the second chip selected is red.

Find $\Pr[E]$, $\Pr[F \mid E]$ and $\Pr[F \cap E]$.

What about $\Pr[F]$?

Probability Tree

Example

Three boxes labeled x, y and z each contain a number of playing cards. Box x contains 2 jacks and 3 kings, box y contains 3 jacks and 1 king, and box z contains 3 kings. An experiment consists of choosing one of the 3 boxes at random and then selecting one of the cards at random from that box.
Draw and label the probability tree for this experiment.
Find the probability that the card selected is a jack from box y.
Find the probability that the card is a king given that the card is selected from box x.
Find the probability that the card selected is a jack.

Exercise #8

A box contains 2 red balls and 3 green balls. A ball is selected at random and its color is noted. If it is red, then it is not replaced in the box and a second ball is drawn and its color is noted. If the first ball is green, then it is replaced in the box, a second ball is drawn at random and its color is noted. Find the probability that the second ball is red.

Exercise #20

Students at GSU are being tested for infection with the North Amazon virus. If a student is infected, then the test on that student is positive (that is, the test indicates the student has the virus) 90 percent of the time. If a student is not infected, then the test on that student is negative (that is, the test says the student does not have the virus) 90 percent of the time. Suppose that 2 percent of the students at GSU are actually infected with the virus. If a student is chosen for testing at random, find the probability that the test is positive.

Probability Tree

Example

A box contains 2 red balls and 3 blue balls. A ball is selected at random, its color noted, and it is replaced. This selection process is repeated 2 more times.

Find the probability that at least 1 blue ball is selected.
Find the probability that at most 1 blue ball is selected.
Find the probability that at least 2 red balls are selected given that at least one ball of each color is selected.
Find the probability that exactly 1 red ball is selected given that the first ball selected is blue.