Chapter 3, Section 2

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Outline

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- Conditional Probability

- Independent Events

Conditional Probability – Using Given Information

Example

A single card is chosen at random from a well-shuffled deck.

- What is the probability the card is a club?
- What is the probability the card is a club if you know the card is a black card?
- What is the probability the card is a club if you know the card is an ace?
- What is the probability the card is a club if you know the card is a heart?

Additional information is important!

conditional Probability

Example

A pair of fair dice, one red and the other green, are rolled.

1. What is the probability that the sum of the dice is 5?
2. Suppose someone else looks at the dice that have been rolled and reports that the red die is 3. Knowing that, what is the probability the sum is 5?
3. What is the probability that the sum is 7?
4. Suppose someone else looks at the dice that have been rolled and reports that the red die is 3. Knowing that, what is the probability that the sum is 7?

Additional information is important!

Conditional Probability

Notation

Let A and B be events in the sample space of an experiment with $\Pr[B] \neq 0$. The **conditional probability of event A given B** is written as

$$\Pr[A \mid B].$$

Definition

$$\Pr[A \mid B] = \frac{\Pr[A \cap B]}{\Pr[B]}.$$

Conditional Probability

Example

A pair of fair dice, one red and the other green, are rolled.

1. Let $A =$ the event that the sum is 5.
   Let $B =$ the event that the red die is 3.
   Let $C =$ the event that the sum is 7.

2. Find $\Pr[A]$

3. Find $\Pr[A \mid B]$

4. Find $\Pr[B \mid A]$

5. Find $\Pr[C]$

6. Find $\Pr[C \mid B]$

Exercise #2

Suppose E and F are events in a sample space S with
$\Pr[E] = .48$, $\Pr[F] = .75$, and $\Pr[E \cap F] = .32$.
Find $\Pr[E \mid F]$ and $\Pr[F \mid E]$.

Exercise #18

A group of 6 students consists of 4 freshmen and 2 sophomores.
Two students are selected simultaneously and at random. Find the
probability that both are freshmen given that at least one is a
freshman.

Exercise

A box contains 3 red poker chips and 4 blue poker chips. Two
chips are selected simultaneously and at random.
Let $A =$ the event that both chips selected are red;
Let $B =$ the event that both chips selected are blue;
Let $C =$ the event that 1 chip is red and 1 chip is blue;
Let $D =$ the event that at least 1 chip is red.

Find $\Pr[A]$, $\Pr[B]$, $\Pr[C]$, $\Pr[D]$, $\Pr[A \mid D]$ and $\Pr[C \mid D]$.

Independent Events

Definition

Let A and B be events in the sample space of an experiment with $\Pr[B] \neq 0$. We have seen examples of when

$$\Pr[\mathbf{A} \,|\, \mathbf{B}] = \Pr[\mathbf{A}],$$

and other examples of when

$$\Pr[\mathbf{A} \,|\, \mathbf{B}] \neq \Pr[\mathbf{A}].$$

We say that A and B are independent events if $\Pr[\mathbf{A} \,|\, \mathbf{B}] = \Pr[\mathbf{A}]$

Note that A and B are independent events if and only if

$$\Pr[\mathbf{A} \cap \mathbf{B}] = \Pr[\mathbf{A}]\Pr[\mathbf{B}].$$

WHY?

Independent Events

Exercise

A pair of fair dice, one red and one green, are rolled.
Let $E =$ the event that the sum of the dice is 6.
Let $F =$ the event that the sum of the dice is 7.
Let $G =$ the event that the green die is odd.
Are E and G independent (events)?
Are F and G independent (events)?
Are E and F independent (events)?

Similar to Exercise #11

Let E and F be events such that $\Pr[E \,|\, F] = \frac{1}{3}$, $\Pr[F \,|\, E] = \frac{2}{5}$, and $\Pr[E \cap F] = \frac{1}{6}$. Are E and F independent?

Exercise

A box contains five balls numbered 1 through 5. An experiment consists of selecting three balls, in succession and without replacement, and noting the number of each ball selected.

Let A = the event that the ball #5 is selected second.

Let B = the event that the ball #1 is selected third.

Let C = the event that the ball #2 is not selected.

Find $\Pr[A]$, $\Pr[B]$, $\Pr[C]$

Are A and C independent (events)?

Are B and C independent (events)?

Are A and B independent (events)?