Chapter 3, Section 1

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Outline

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- General Experiments and Sample Spaces
- Properties of Probability

A probability measure assigns to each event E of a sample space S a number $\Pr[E]$ that is an indication of the likelihood the outcome of the experiment is in E.

This assignment of probabilities must satisfy the following axioms (assumptions):

- $0 \leq \Pr[A] \leq 1$, for each event A of the sample space S.
- $\Pr[S] = 1$
- If E and F are disjoint events in S, then

$$\mathbf{Pr}[\mathbf{E} \cup \mathbf{F}] = \Pr[\mathbf{E}] + \Pr[\mathbf{F}] \, .$$

Properties

Additional properties of a probability measure:

1. For any $E \subset S$, $\Pr[\mathbf{E}'] = \mathbf{1} - \Pr[\mathbf{E}]$.
2. If $E_1, E_2, \ldots, E_k$ are pairwise disjoint events, then

$$\Pr[\mathbf{E_1} \cup \mathbf{E_2} \cup \cdots \cup \mathbf{E_k}] = \Pr[\mathbf{E_1}] + \Pr[\mathbf{E_2}] + \cdots + \Pr[\mathbf{E_k}] \, .$$

3. For any events A and B in S,

$$\Pr[\mathbf{A} \cup \mathbf{B}] = \Pr[\mathbf{A}] + \Pr[\mathbf{B}] - \Pr[\mathbf{A} \cap \mathbf{B}] \, .$$

Example

An experiment is to toss a fair coin until either you get a Tail (T) or the coin is tossed 4 times. Draw a tree diagram to help find the sample space S. How many outcomes does this experiment have? How should we assign probability to the individual outcomes?

Exercise

Exercise #2

Let A and B be events in a sample space S, and assume $\Pr[A] = .45$, $\Pr[B] = .75$ and $\Pr[A' \cap B] = .35$. Find

(a) $\Pr[B' \cap A] =$

(b) $\Pr[A \cap B] =$

(c) $\Pr[A' \cap B'] =$

(d) $\Pr[A \cup B] =$

Example

The sample space for an experiment is $S = \{v, w, x, y, z\}$. Suppose that

- $\Pr[\{v, x, z\}] = .40$,
- $\Pr[\{x\}] = .12$,
- $\Pr[\{v\}] = \Pr[\{z\}]$ and
- $\Pr[\{w\}] = 2\Pr[\{y\}]$.

Find $\Pr[\{w, y, z\}]$.

Exercise

Exercise #8

Suppose E, F and G are events in a sample space S, with $\Pr[E] = .45$, $\Pr[F] = .5$, $\Pr[G] = .5$, $\Pr[E \cap F] = .2$, $\Pr[E \cap G] = .3$, $\Pr[F \cap G] = .25$, and $\Pr[E \cap F \cap G] = .05$.

Find

(a) $\Pr[E' \cap F' \cap G] =$

(b) $\Pr[E \cup G] =$

(c) $\Pr[E \cup F \cup G] =$