Chapter 2, Section 3

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Outline

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- Combinations
- Counting Combinations
- When do I multiply? When do I add?

Combinations

- A combination is a subset (or an unordered collection) of distinct elements selected from a given set X. The size of the combination is the number of elements in the subset that is selected.
- For a given set X with $n(X) = n$ we want to count how many different combinations of size k can be selected from X. We denote this number $\mathbf{C(n, k)}$.

Example

- $X = \{a, b, c, d\}$ The combinations of size 2 selected from X are $\mathbf{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}}$.
  We see $\mathbf{C(4, 2) = 6}$.
- In how many ways can we choose 2 people from the group of friends Alice, Bob, Carol, and David to drive to Pizza Hut to buy a pizza?

Counting Combinations

- Let X be a set with $n(X) = n$. An experiment is to select k objects from X and arrange them in a particular order. This can be thought of as a 2 stage experiment.
  1. Stage 1 Select a subset of k objects from X.
  2. Stage 2 Arrange the objects selected in Stage 1 in some particular order.

How many outcomes does this experiment have? $\mathbf{P(n, k)}$
Stage 1 has $\mathbf{C(n, k)}$ outcomes.
Stage 2 has $\mathbf{k!}$ outcomes.
By the Multiplication Principle, $\mathbf{P(n, k) = C(n, k) \times k!}$

Counting Combinations

$$\mathbf{C(n,k) \times k! = P(n,k)}$$

Divide both sides by $k!$ and

$$\mathbf{C(n,k) = \frac{P(n,k)}{k!} = \frac{\frac{n!}{(n-k)!}}{k!} = \frac{n!}{(n-k)!\ k!}}\ .$$

$C(4,2) = \frac{4!}{(4-2)!\ 2!} = \frac{4!}{2!\ 2!} = \frac{4\times 3\times 2\times 1}{2\times 2} = 6$

Counting Combinations

A chess club has 7 members. In how many ways can they select a group of 3 of the members to go to a tournament?

An ice cream store sells ice cream cones in 10 different flavors. A customer is going to purchase 4 cones, each of a different flavor? In how many ways can she do this?

Exercise #10: A three card hand is dealt from a standard deck.

- How many such 3-card hands are there?
- How many such 3-card hands are all spades?
- How many such 3-card hands are all of the same suit?
- How many such 3-card hands are all of the same rank?

When do I multiply? When do I add?

[Examples]

1. A box contains 7 red balls numbered 1 through 7.
   - How many different sets of 3 balls can be selected from the box?
   - How many different sets of at least 4 balls can be selected from the box?
   - If the balls are selected one after another without replacement in how many ways can 3 balls be selected?
2. A sorority has 20 members, 5 are sophomores, 7 are juniors and 8 are seniors. A group of 6 is selected to go to a workshop.
   - How many groups of 6 can be selected?
   - How many groups of 6 can be selected if 2 must be sophomores, 1 must be a junior and 3 must be seniors?

When do I multiply? When do I add?

[Exercise #22]

A security entry lock consists of a keyboard with the numbers 1 through 9. An "entry code" consists of three single numbers to be punched in order, or two single numbers punched in order followed by one pair of numbers (a pair is two different numbers), the pair to be punched simultaneously. How many different entry codes are there?

A group of 9 friends consists of 6 women and 3 men. They are going to a restaurant and have a van that holds 5 people and a car that holds 4 people. In how many ways can they split the group into a "van group" and a "car group"? In how many ways can they do this if there must be exactly 4 women in the van? In how many ways can they do this if there must be at least 3 women in the van?