Chapter 2, Section 2

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Fall 2014

Outline

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- Permutations
- Counting Permutations
- Counting More General Arrangements

Permutations

- A permutation is an ordered list (or ordered arrangement) of distinct elements selected from a set.
- A permutation can be thought of as an outcome of a multistage experiment that consists of selecting elements from a set, one after the other **without replacement**. The length of the permutation is the number of stages in the experiment or equivalently the number of elements in the list.

Examples

- Write down all the permutations of length 2 selected from the set $\{w, x, y, z\}$.
- In how many ways can we form a lineup of 4 persons selected from the set {Alice, Bob, Carol, David, Erica, Frank, George}?

Counting Permutations

Examples

1. In how many different ways can we take a standard deck of 52 cards and turn up 7 of the cards in a row on a table? Please focus on how we get the answer and not on the actual number itself.
2. Eight high schools are entered in a cross country race. If there are no ties, then how many different orders of finish are possible in the team competition?

Counting Permutations

Permutation Principle

The number of permutations of length k using elements from a set of n distinct objects is denoted by $P(n, k)$. (This is also sometimes read "the number of permutations of n things taken k at a time.")

$$P(n,k) = n \times (n-1) \times (n-2) \times \cdots \times (n-k+1)\,.$$

In many of these counting problems we end up computing the product of all the consecutive positive integers starting at some number and decreasing down to 1. The following notation has been developed. ($n!$ is read as "n **factorial**".)

$$\mathbf{n! = n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1}$$

Formula for $P(n, k)$

$$\begin{aligned}
P(n,k) &= n \times (n-1) \times \cdots \times (n-k+1) \\
&= \frac{n \times (n-1) \times \cdots \times (n-k+1) \times (n-k)!}{(n-k)!} \\
&= \frac{n \times (n-1) \times \cdots \times (n-k+1) \times (n-k) \times \cdots \times 2 \times 1}{(n-k)!} \\
&= \frac{n!}{(n-k)!}
\end{aligned}$$

[Examples]

1. $P(5,3) = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 5 \times 4 \times 3 = 60$
2. $0! = 1$ (This is a special definition.)
3. $P(7,7) = \frac{7!}{(7-7)!} = \frac{7!}{0!} = \frac{7!}{1} = 7! = 5040$

[Examples]

1. In how many different ways can we take a standard deck of 52 cards and turn up 7 of the cards in a row on a table?
   $P(52, 7) = \frac{52!}{(52-7)!} = \frac{52!}{45!} = 52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47 \cdot 46$
2. Eight high schools are entered in a cross country race. If there are no ties, then how many different orders of finish are possible in the team competition?
   $P(8, 8) = \frac{8!}{(8-8)!} = \frac{8!}{0!} = \frac{8!}{1} = 8! = 40,320$

[Exercise #12]

How many different 4 letter "words" can be formed using letters in the word "formula"? How many of them contain the letter m? How many of them do not contain the letter m?

Exercises

[Exercise #16]

A committee has 9 members consisting of 5 men and 4 women. Three different members are to be given special tasks: one is to serve as a delegate to a convention, one is to seek new members, and one is to seek donations. In how many different ways can the three tasks be assigned so that at least one man and at least one woman are assigned tasks?

Allowing Some Repetition

How many 7-letter words can be formed from the letters of the word "LESSONS"?

If we assume first that the 3 repetitions of the letter S are different, say LESSONS, then there would be $P(7,7) = 7!$.

Of course, the 3 occurrences of the letter S are not different. This means that each **particular** arrangement of the letters of the word "LESSONS", such as NSOLSES, is counted 3! times.

NSOLSES, NSOLSES, NSOLSES,
NSOLSES, NSOLSES, NSOLSES.

So, to remove the "over-counting" of distinct arrangements we need to divide by 3!

There are $\frac{7!}{3!} = 7 \times 6 \times 5 \times 4 = 840$ different 7-letter words that can be formed from the letters of the word "LESSONS".

Example

How many 6-letter words can be formed from the letters of the word "better"?

How many 7-digit numbers can be formed from the digits of the number 8384378?