Chapter 2, Section 1

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Outline

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- Events
- Probability Properties
- Probability Assignments
- Equally Likely Experiments

Events

If S is the sample space for an experiment, then any subset A of S is called an event. (Each element of an event A is an outcome of the experiment.)

Examples

- A fair coin is tossed three times. Let E be the event that exactly two of the three tosses is a head (H). Find E.
- Same experiment as above. Let A denote the event that at most two of the three tosses are heads and let B be the event that at least two of the three tosses are heads. Find $A \cap B$ and $A \cup B$.

Probability Assignments

The Idea

The probability of a set E of outcomes in a sample space S of an experiment is a number that represents the likelihood that one of the outcomes in E will occur when the experiment is performed.

- We denote this number by $\Pr[E]$. It is always the case that $0 \leq \Pr[E] \leq 1$.
- The closer $\Pr[E]$ is to 1 the more likely it is that the outcome will be in E.
- The closer $\Pr[E]$ is to 0 the less likely it is that the outcome will be in E.

Probability Assignments

What Must Be True

Suppose that an experiment has exactly n outcomes $\mathcal{O}_1, \mathcal{O}_2, \ldots, \mathcal{O}_n$.
That is, the sample space is $S = \{\mathcal{O}_1, \mathcal{O}_2, \ldots, \mathcal{O}_n\}$. If the probabilities are $\Pr[\mathcal{O}_1] = w_1$, $\Pr[\mathcal{O}_2] = w_2$, ..., $\Pr[\mathcal{O}_n] = w_n$, then

1. For each i, $0 \leq w_i \leq 1$ and
2. $w_1 + w_2 + \cdots + w_n = 1$.

[Exercise #4]

An experiment with 5 outcomes has $w_1 = .23$, $w_2 = .14$, $w_3 = w_4$ and $w_5 = .29$.
Find w_4.

Probability Properties

If S is the sample space for an experiment and $E \subset S$ is any event, then the probability of E, denoted $\Pr[E]$ is the sum of the probabilities of all the outcomes that belong to E.
A consequence of this is that $\Pr[E] + \Pr[E'] = 1$.

[Exercise #6]

Let $S = \{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4\}$ be a sample space. Suppose that $\Pr[\{\mathcal{O}_1, \mathcal{O}_3\}] = .55$ and $\Pr[\{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_4\}] = .75$.
Find $\Pr[\{\mathcal{O}_2, \mathcal{O}_4\}]$. Find $\Pr[\{\mathcal{O}_1\}]$.

Methods of Assigning Probability

Relative Frequency Method

The experiment is performed many (thousands? millions?) of times and we observe what fraction of the time a particular outcome occurs. This fraction is assigned as the probability of that outcome.

To know how realistic this assignment is we need to have a good understanding of statistics.

Examples

- There is a 30% chance of rain on campus this afternoon. That is, the probability of rain this afternoon on campus is .30.
- A flu vaccine is given to a child. The probability the child will not get the flu is .05.

Methods of Assigning Probability

Deductive Method

We assign probabilities to outcomes of an experiment by making assumptions and reasoning about the experiment.

- We randomly select 1 poker chip from a box that contains 3 red, 2 white and 5 blue poker chips and note its color. The probability the chip is red is $3/10$.
- We roll a pair of fair dice and write down the sum of the numbers that show on the dice. How might we determine the probability that the sum is 5?

Equally Likely Experiements

An experiment is called an equally likely experiment if each of its possible outcomes is assigned the same probability.
This assignment is done using the deductive method.

- A single card is dealt from a well shuffled deck and the rank and suit noted.
- A fair die is rolled and the number showing is recorded.

Equally Likely Experiements

[Exercise #26]

Of the 200 employees at Everest Securities, Inc., 65 are in retail sales, 45 are in institutional sales, 25 are in research, and the remainder are in administration. An experiment consists of selecting an employee at random and noting whether the employee is in sales. What probabilities should be assigned to the outcomes on the basis of the data?

[Exercise]

A box contains balls numbered 1 through 4. Two balls are selected at random in succession without replacement and the number on each is recorded. How many outcomes are there? How should we assign probabilities to these outcomes? If B is the event that the sum of the two numbers is at least 5, what is $\Pr[B]$?