

# CHAPTER 1, SECTION 4

Doug Rall  
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## Outline

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- Experiments and Sample Spaces
- Tree Diagrams
- The Multiplication Principle

In this course an **experiment** is a procedure whose performance results in one of some finite number of outcomes. Unless there is only one possible outcome to the experiment, the particular outcome cannot be known with 100% accuracy **before** the performance of the experiment.

## Examples

- The top card of a well-shuffled deck is turned face up on the table and its suit is noted.
- A fair coin is tossed and the side up is noted – either heads (H) or tails (T).
- A subset of 2 poker chips is selected from a box that contains 3 red, 4 white and 3 blue poker chips.

# Sample Space

## Definition

A **sample space** for an experiment is a set that has as its elements all the possible outcomes of the experiment.

**Example:** If the top card of a well-shuffled deck is turned face up on the table and its suit is noted, then the sample space is  $S = \{\text{Hearts, Spades, Clubs, Diamonds}\}$ .

**Example:** An experiment is to roll a fair die two times and add the results. The sample space is  $X = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ .

Many of the experiments we will consider consist of several **stages** or **steps** performed one after the other. To help construct the sample space we will make a **tree diagram**.

**Example:** A card is selected at random from a standard deck of cards, its suit is noted and it is not returned to the deck. A second card is selected and its suit is noted. What is the sample space?

**Example:** There are three boxes on a table labeled *I*, *II* and *III*. Box *I* contains a red marble and a blue marble; Box *II* contains a white marble and a blue marble; Box *III* contains a red marble. An experiment consists of selecting a box and then selecting a marble from that box. Determine the sample space.

## Exercises

### Exercise #4

A cooler contains 2 cans of diet cola and 4 cans of regular soda. An experiment consists of selecting cans one after another **without replacement**, and noting the type of soda of each selection until two consecutive cans of the same type of cola or a total of four cans are selected. Let  $S$  be the sample space. Find  $S$  and  $n(S)$ .

### Exercise

Alice, Bob, Carol and David have all shown up at the driver's license testing station to take their driver's test. The person giving the test will take them one at a time to test their driving skills. In how many different orders can the driving tests be given?

# The Multiplication Principle or Product Rule

## 2 stages

Consider an experiment that has 2 stages (one after the other). Suppose the first stage has  $n_1$  possible results. Also, suppose that regardless of the result of the first stage there are  $n_2$  possible results from the second stage. The experiment has a total of  $n_1 \times n_2$  outcomes.

## Multistage

An experiment has  $t$  stages.

- The first stage has  $n_1$  possible results.
- Regardless of the particular result in first stage, the second stage has  $n_2$  possible results.
- For each  $i$  the following is true: Regardless of the particular results of the previous stages, the  $i^{\text{th}}$  stage has  $n_i$  possible results.

The entire experiment has  $n_1 \times n_2 \times \cdots \times n_t$  outcomes.

## Exercises

### Exercise #2

A multiple-choice test has 10 questions, and there are 5 choices for the answer to each question. An answer sheet has 1 answer marked for each question. How many different answer sheets are possible? How many different answer sheets are possible if questions may be left unanswered? [Does the multiplication principle apply here?]

### Exercise #12

A coin is tossed repeatedly, and the result (H or T) of each toss is recorded. The experiment stops if there are two consecutive tosses with the same result or if the coin is tossed three times. How many possible outcomes are there for this experiment? [Does the multiplication principle apply here?]

## Exercise #26

A box contains 1 red, 1 white, and 2 green balls. An experiment consists of drawing balls in succession without replacement and noting the color of each until a red ball is drawn.

- (a) Draw a tree diagram for this experiment and give the sample space  $S$ . Then find  $n(S)$ .
- (b) Give the subset  $E$  of  $S$  that consists of all outcomes of this experiment in which a white ball is not drawn.

## Exercise #32

A box contains 3 green balls, 1 white ball, and 2 blue balls. An experiment consists of selecting three balls, one after another without replacement, and noting the color of each ball selected. Find the number of outcomes in the sample space of this experiment.