Chapter 1, Section 2

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Outline

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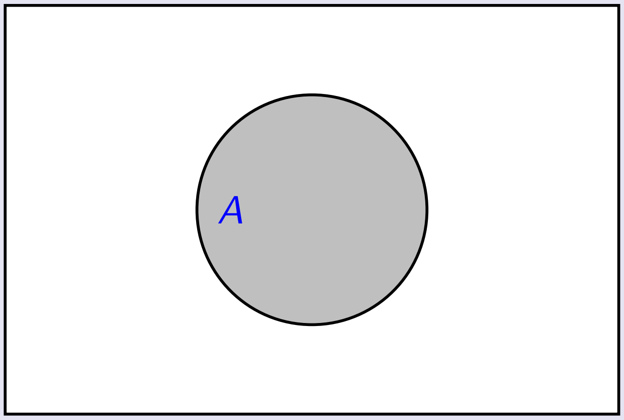
- Venn Diagrams
- Representing Subsets Using Venn Diagrams
- deMorgan's Laws
- Associative & Distributive Laws
- Partitions
- Number of Elements and Partition Principle

Venn Diagrams

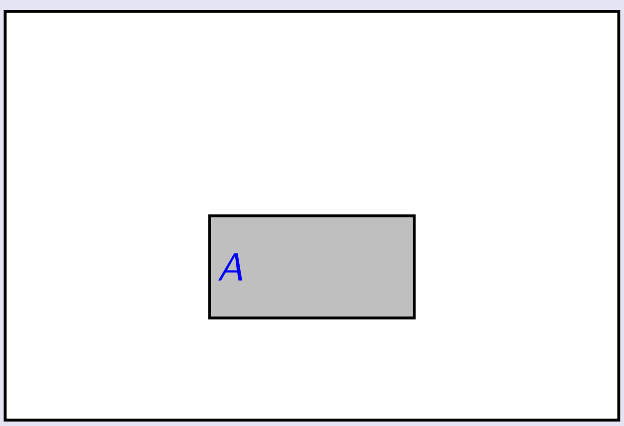
Representing the universal set U and one subset $A \subset U$



U: the region inside the rectangle
A: the region inside the circle

Venn Diagrams

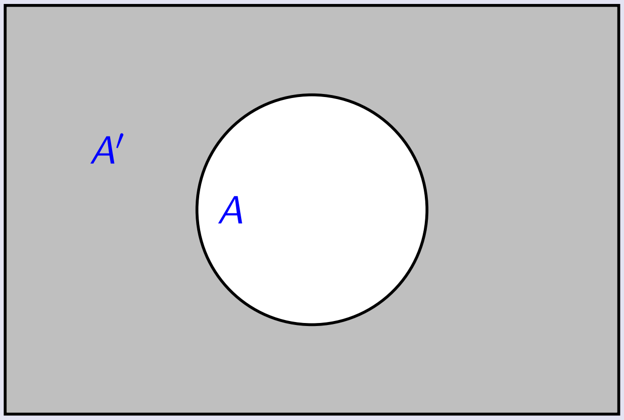
Representing the universal set U and one subset $A \subset U$



U: the region inside the rectangle
A: the region inside the smaller rectangle

Venn Diagrams

Representing the complement of a set

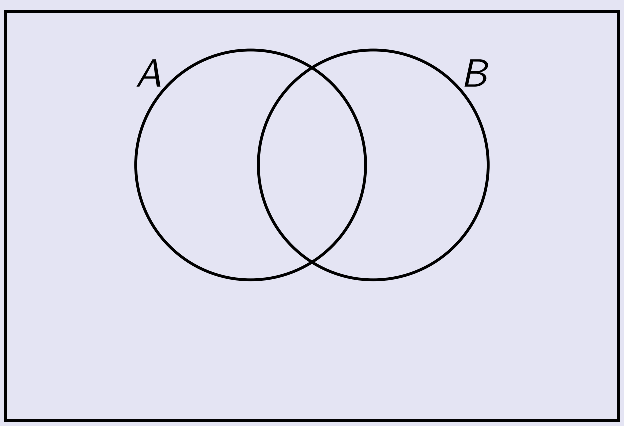


A: the region inside the circle

A': the region that is both outside the circle **and** inside U

Venn Diagrams

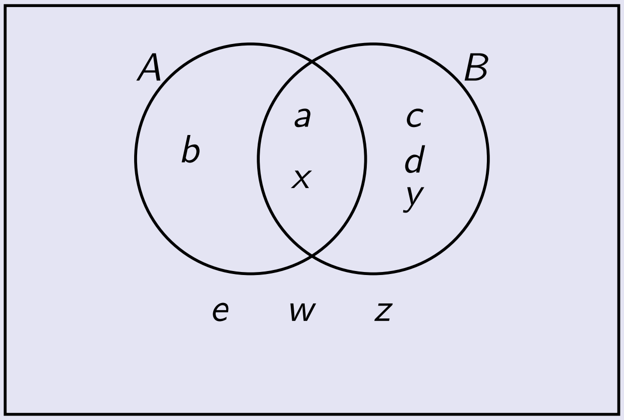
Two subsets A, B in "standard position"



- Shade the subset $A \cup B$
- Shade the subset $A \cap B$
- Shade the subset $A \cap B'$
- Shade the subset $B \cup A'$

Venn Diagrams

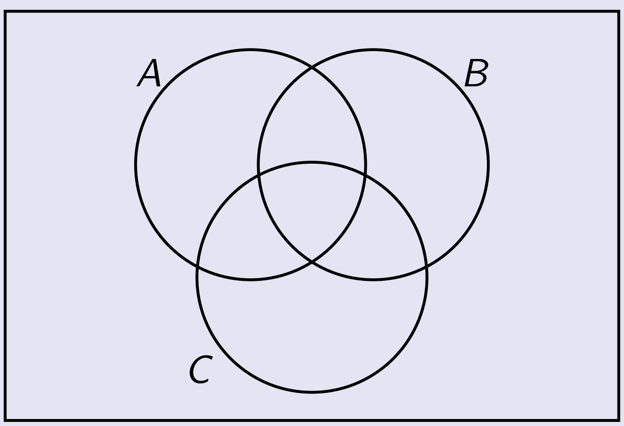
Exercise



- $A =$
- $B =$
- $A \cap B' =$
- $B' \cup (A' \cap B) =$

Venn Diagrams

Three subsets A, B and C in "standard position"



- Shade the subset $A \cup B$
- Shade the subset $A \cap B \cap C$
- Shade the subset $A \cap B' \cap C'$
- Shade the subset $(B \cap A') \cap C$
- Shade the subset $(B \cap A') \cup C$

Two Laws

DeMorgan's Laws

For a universal set X and $A \subset X$, $B \subset X$

$(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

Associative and Distributive

For a universal set X and $A \subset X$, $B \subset X$ and $C \subset X$

$(A \cup B) \cup C = A \cup (B \cup C)$

$(A \cap B) \cap C = A \cap (B \cap C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Partitions

Definition

A list of sets $A_1, A_2, \ldots, A_k$ are pairwise disjoint if **every** pair of them are disjoint. (that is, every pair of them have empty intersection)

Example

$\{1, 3, 5\}$, $\{2, 4, 6\}$, $\{7, a, z\}$, $\{b, c\}$ are pairwise disjoint

$\{1, a, b\}$, $\{2, 4\}$, $\{3, b\}$ are not pairwise disjoint even though $\{1, a, b\} \cap \{2, 4\} \cap \{3, b\} = \emptyset$.

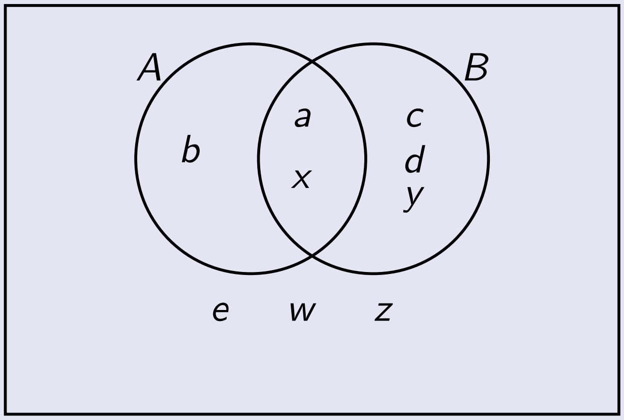
Definition

A partition of a set X is a collection of nonempty subsets of X such that

- the subsets are pairwise disjoint, and
- the union of the subsets is X.

Partitions

Examples

- $\{1,3,5,7\}, \{2,4\}, \{6\}$ is a partition of $\{1,2,3,4,5,6,7\}$
- $\{(1,b)\}, \{(2,a),(1,a),(3,b)\}, \{(2,b),(3,a)\}$ is a partition of $\{1,2,3\} \times \{a,b\}$.
- 

  $A \cap B'$, $A \cap B$, $A' \cap B$, $(A \cup B)'$ is a partition of $\{a,b,c,d,e,w,x,y,z\}$.

Partition Principle

Notation

Let S be a set with a finite number of elements. We denote the number of elements in S by $n(S)$.

Examples

$n(\{a,b,c,d,e,f,g\}) = 7$

$n(\{x,y\} \times \{7,8,9\}) = 2 \cdot 3 = 6$. (General Principle)

$n(\emptyset) = 0$

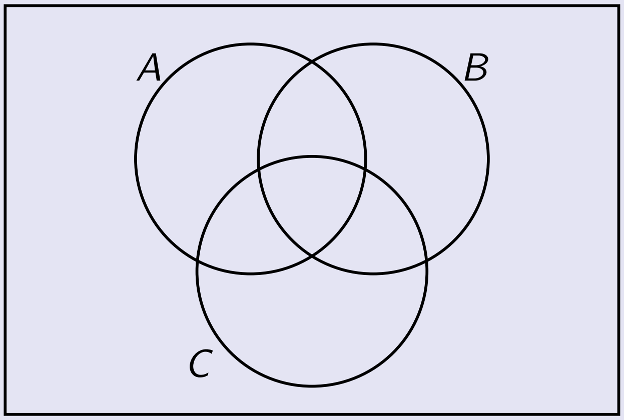
$n(\{1,2,\{4,5\},3\}) = 4$

Partition Principle

If $A_1, A_2, \ldots, A_k$ is a partition of a finite set X, then $n(X) = n(A_1) + n(A_2) + \cdots + n(A_k)$.

Counting Principles

Partition Principle



The sets $A \cap B \cap C$ and $A \cap B' \cap C$ and $A \cap B \cap C'$ and $A \cap B' \cap C'$ partition the set A.

By the partition principle, $n(A) =$

Exercises

[#12] Let U be a universal set with disjoint subsets A and B.
$n(U) = 55$, $n(A) = 25$, and $n(B) = 10$.
Find $n(A' \cup B)$.
Find $n(A' \cap B)$.
Find $n((A \cup B)')$.

Exercises

[#28] A set X with $n(X) = 45$ is partitioned into three subsets X_1, X_2, and X_3. If $n(X_2) = 2n(X_1)$ and $n(X_3) = 3n(X_2)$, find the number of elements in X_1.

Exercises

[#22] Let A, B and C be subsets of a universal set U with A and B disjoint, $n(U) = 110$, $n(A) = 35$, $n(B) = 44$, $n(A \cup B \cup C) = 96$, and $((A \cup B) \cap C) = 28$. Find $n(C)$.

- Draw a Venn diagram representing the given information.
- Fill in any additional information.
- The following 4 subsets partition U: A, B, $C \cap A' \cap B'$ and $A' \cap B' \cap C'$