Chapter 1, Section 1

Doug Rall
Fall 2014

Outline

Outline

- Sets and set notation
- Subsets
- Union
- Intersection
- Universal set
- Cartesian product

Sets and set notation

Definition

A set is a collection of distinct objects. The objects in a set are called its elements.

- We typically use upper case letters (such as A, E, S or X) to denote a set and lower case letters (r, x, y, $\ldots$) to denote the elements of a set.
- For a generic set X we write $a \in X$ (read "a is an element of X") to mean that a is one of the objects in the set X.
- To indicate that a particular object c is not one of the objects in a set A we write $c \notin A$ (read "c is not an element of A")
- The only important thing about a set is which objects it contains. So, when are two sets equal?

Specifying a set

Two ways to specify a set

1. For sets with few elements we can list the elements, separated by commas, and enclosed in set brackets, $\{ \quad \}$.
   - $A = \{t, u, v, w, x, y, z\}$
   - $D = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$
2. Using a rule to describe the elements ("set-builder" notation) that belong to the set.
   - $D = \{x : x \text{ is an integer between 2 and 10 inclusive}\}$
     Read "the set of all x such that x is an integer between 2 and 10 inclusive."
   - $F = \{s : s \text{ is a current Furman student}\}$

Subsets

Definition

If R and S are sets, then R **is a subset of** S if every element in R is also an element in S. We write $R \subset S$ or $R \subseteq S$ to indicate that R is a subset of S. ($\not\subset$ means ...)

Examples

$\{1,3,5,7,9\} \subset \{1,2,3,4,5,6,7,8,9,10\}$
Let $A = \{x : x \text{ is a Furman student from Utah}\}$
and $F = \{s : s \text{ is a Furman student}\}$
$A \subset F$
$\{1,3,5,7\} \not\subset F$
$\{1,2,3,4,5,6,7,8,9,10\} \not\subset \{1,3,5,7,9\}$
T/F: $\{1,2,3,4\} \subset \{1,2,3,4\}$.

Sets and Subsets

Examples

Let $X = \{a,b,c,d,e,f,g,h,x,y,z\}$

1. T/F: $b \in X$ True
2. T/F: $11 \in X$ False
3. T/F: $\{a,c,z\} \subset X$ True
4. T/F: $\{x,y,z\} \in X$ False
5. T/F: $d \subset X$ False

Union of Sets

Definition

Let R and S be sets. The union of R and S is a set, denoted by $R \cup S$, and defined by

$$R \cup S = \{z \,|\, z \in R \text{ or } z \in S\}.$$

Examples

[Ex #4] Let $E = \{2, t, y\}$ and $F = \{1, 2, u, x, y\}$.

- $E \cup F = \{2, t, y, 1, u, x\} = \{1, 2, t, u, x, y\}$
- $F \cup F = F$

Intersection of Sets

Definition

Let R and S be sets. The intersection of R and S is a set, denoted by $R \cap S$, and defined by

$$R \cap S = \{z \,|\, z \in R \text{ and } z \in S\}.$$

Examples

[Ex #4] Let $E = \{2, t, y\}$ and $F = \{1, 2, u, x, y\}$.

- $E \cap F = \{2, y\}$
- $F \cap F = F$
- T/F: If A and B are any sets, then $A \subset (A \cup B)$. True
- T/F: If A and B are any sets, then $A \subset (A \cap B)$. False
- T/F: If A and B are any sets, then $(A \cap B) \subset A$. True

Definition

1. The empty set is a set that contains no elements. The empty set is denoted by $\emptyset$.
   - $\{a, b, c, d\} \cap \{1, 2, 3, x, y, z\} = \emptyset$
   - $F =$ the set of all current Furman students.
     $A =$ the set of all current Furman students from Iceland.
     Is $A = \emptyset$?
2. Two sets R and S are called disjoint if $R \cap S = \emptyset$.

Definition

In a typical problem all the objects (elements) being considered belong to one large set X. Such a set X is called the universal set for that problem. If $E \subset X$, then the complement of E is the set $E' = \{a \in X \mid a \notin E\}$.

- Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ be universal set.
  $\{1, 3, 5\}' = \{2, 4, 6, 7, 8, 9, 10\}$

Problems

- If R and S are subsets of a universal set U, how would you use set notation to write the set of all elements that belong to R but not to S?
- How would you write the set of all elements that belong to R or to S but not to both?

#10 $U = \{x, y, z, 1, 2, 3\}$, $A = \{y, z, 2\}$, $B = \{y, 1, 2\}$, $C = \{x, 3\}$

(a) $A \cup B =$
(b) $B \cap C =$
(c) $(A \cup B) \cap (B \cup C) =$
(d) $(B \cap A') \cap C' =$
(e) $(B \cup C)' \cap A =$

Cartesian Product

Definition

If R and S are sets, then the Cartesian product of R and S is the set denoted by $R \times S$ and defined by
$R \times S = \{(a, b) \mid a \in R \text{ and } b \in S\}$.

Examples

Let $A = \{3, 6\}$ and $B = \{r, s, t\}$.
$A \times B =$
$B \times A =$
$A \times A =$
Do there exist sets E and F such that
$E \times F = \{(a, 1), (a, 3), (b, 1), (a, 2), (b, 3)\}$?