Topics in Graph Theory
Graphs and Their Cartesian Product

Corrections and Improvements

* The subject index entry “uniquely distinguishable, 146” should not be a subentry of “undominated.”

* page 175, line 6: “Let P a shortest path” should be “Let P be a shortest path”

* page 106, Proposition 13.2: The proof can be considerably shortened from that given in the book.

The following paragraphs contain the statement of the proposition and a new, shorter proof. The notation is compatible with that of Figures 13.1 and 13.2.

For this proof we introduce the notation G-*edge* for the edges that are in G-fibers and H-*edge* for those in H-fibers.

Proposition 13.2 *A connected subgraph W of $G \,\square\, H$ is a box of $G \,\square\, H$ if and only if for any two adjacent edges e and f of W that are in different fibers, the unique square of $G \,\square\, H$ that contains e and f is also in W.*

Proof. Every connected box W satisfies the conditions of the proposition, even if it is a subgraph of a fiber.

Conversely, let the condition of the proposition be satisfied. Consider two vertices $u = (u_1, u_2)$ and $y = (y_1, y_2)$ in W and a u,y-path P in W. We show first that there exists a u,y-path Q in W where the H-edges precede the G-edges.

Suppose there are two successive edges e and f' in P, where $e = (g, h)(g', h)$ is a G-edge and $f' = (g', h)(g', h')$ an H-edge. If we replace e by $e' = (g, h)(g, h')$ and f' by $f = (g, h')(g', h')$ we obtain another

u,y-path, say P', which is also in W since e, f' are in different fibers. Continuing in this way we end with a u,y-path $Q \subseteq W$ in which the H-edges precede the G-edges.

This implies that $(u_1, y_2) \in V(W)$. Similarly one shows that $(y_1, u_2) \in V(W)$, and hence $V(W) = p_G(V(W)) \times p_H(V(W))$. If W is induced, say if it is convex, then we are through.

Otherwise, suppose there is an edge e such that $p_G(e) \in p_G(W)$, but $e \notin W$. By the above there is an $f \in W$ with $p_G(f) = p_G(e)$. Set $e = xy$ and $f = wz$ with $p_G w = p_G x$. Then there is a path Q' of H-edges from w to x. If the path has length one, then an application of the square property shows that $e \in W$, otherwise we use induction with respect to the length of Q'.

Similarly one treats the case $p_H(e) \in p_H(W)$, but $e \notin W$. $\square$