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- 1887 -1920

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Hardy and Ramanujan

- Hardy has written that Ramanujan was then nearly twenty-five. The years between eighteen and twenty-five are the critical years in a mathematician's career, and the damage had been done. Ramanujan's genius never had again its chance of full development. (Ramanujan by G. H. Hardy, Cambridge, 1940, page 6)

Concerning the results in the initial letter from Ramanujan, Hardy said: A single look at them is enough to show that they could only be written down by a mathematician of the highest class. They must be true because, if they were not true, no one would have had the imagination to invent them. (Ramanujan by G. H. Hardy, Cambridge, 1940, page 9)
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Patterns

► Hardy (page 2027): “A mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas.”
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▶ What does Hardy mean when he says there is “no permanent place for ugly mathematics?”
Aesthetic appeal

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- Does his argument work?
Trivial mathematics

- Hardy claims (page 2029) that a chess problem is mathematics, but trivial mathematics. In what sense is chess “trivial” mathematics?
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Note: he considers a mathematical theorem “serious,” not for practical reasons, but for the significance of the ideas which it connects.

How does the beauty of a theorem compare with the beauty of a poem?
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Two examples

- Hardy wishes now to illustrate what he has been describing by considering two mathematical theorems. What restrictions in choice is he working under?

- Reductio ad absurdum (argument by contradiction) (page 2031): “A far finer gambit than any chess gambit: a chess player may offer the sacrifice of a pawn or even a piece, but a mathematician offers the game.”
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Theorem and proof

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- the Pythagorean result easily extends to other results, led to the realization that rational arithmetic wasn’t enough, and prompted the development of the theory of proportions and irrational numbers.

But are these theorems practical? Does an engineer need that many primes or that many decimal places?
Usefulness of mathematics

In another part of the *Apology*, Hardy argues that “[t]he ‘real’ mathematics of the ‘real’ mathematicians, the mathematics of Fermat and Euler and Gauss and Abel and Riemann, is almost wholly ‘useless’ (and this is as true of ‘applied’ as of ‘pure’ mathematics).”
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- Another exception: he is known in biology for the Hardy-Weinberg Law for genetic equilibrium.
Elsewhere in his *Apology*, Hardy explains his position on mathematical realism:

> I believe that mathematical reality lies outside us, that our function is to discover or observe it, and that the theorems which we prove, and which we describe grandiloquently as our 'creations', are simply our notes of our observations.
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Geometry is not about the physical world: “earthquakes and eclipses are not mathematical concepts.” (page 2035)
Mathematicians offer a supply of models with which to approximate the physical world: pure mathematics is independent of any “detail of the physical world,” (page 2035) whereas the applied mathematician offers the natural scientist models from which to choose. Yet “no mathematician is so pure that he feels no interest in the physical world.” (page 2036)
Mathematical realism (cont’d)

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In a passage not included here, Hardy explains how mathematical objects are more real than physical objects: a prime number is what it is, independent of anything we might think about it, but a physical object, such as a chair, is nothing like how it appears to us.
In the end, Hardy believes that mathematics must be defended on aesthetic grounds. As for his own life:
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The case for my life, then, or for that of any one else who has been a mathematician in the same sense in which I have been one, is this: that I have added something to knowledge, and helped others to add more; and that these somethings have a value which differs in degree only, and not in kind, from that of the creations of the great mathematicians, or of any of the other artists, great or small, who have left some kind of memorial behind them. (page 2038)