The Rhind Papyrus

Mathematics 15: Lecture 2

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September 13, 2006

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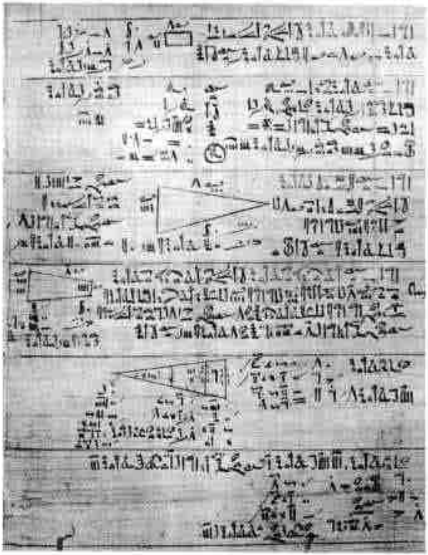
- Scholarly work? Manual? Lesson book?

Part of the Rhind papyrus



Example: multiplication

- Ahmes would compute $45 \times 53 = 2385$ as follows:

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	2	106
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Total	45	2385

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- Note: this follows from

$$45 \times 53 = (1 + 4 + 8 + 32) \times 53 = 1 \times 53 + 4 \times 53 + 8 \times 53 + 32 \times 53.$$

Example: division

- Ahmes would compute $437 \div 23 = 19$ as follows:

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- But: Egyptians insisted on fractions with unit numerator (except for $\frac{2}{3}$).

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- Example: $\frac{2}{61}$ was written as $\frac{1}{40}$, $\frac{1}{244}$, $\frac{1}{488}$, $\frac{1}{610}$. Why not as $\frac{1}{61}$, $\frac{1}{61}$?

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  - So we want:

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  - Hence $y = \frac{5}{3}$, $x = \frac{55}{6}$, and the shares are

$$\frac{5}{3}, \frac{65}{6}, 20, \frac{175}{6}, \frac{115}{3}.$$

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  - Now the sum of the shares is $5+10x = 60$, but we want it to be 100. So multiply by $\frac{100}{60} = \frac{5}{3}$.
  - Thus the shares are $\frac{5}{3}$, $\frac{65}{6}$, 20, $\frac{175}{6}$, and $\frac{115}{3}$.

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  - Hence $\frac{11}{2}$ should work.

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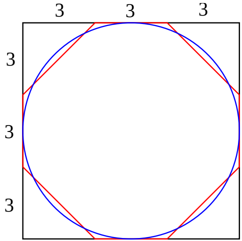
  - *Real numbers*: set of all rational and *irrational* numbers (such as $\sqrt{2}$ and π)

Problems

1. Compute 25×35 using the Egyptian method.
2. Compute $414 \div 18$ using the Egyptian method.

Problems (cont'd)

3. The Rhind papyrus appears to calculate the area of a circle as follows: Draw a square with each side 9 units in length. Divide the sides of the square in thirds and inscribe an octagon in the square by cutting off the corners of the square as shown in the figure below.



   a. Show that the area of the octagon is 63 square units.
   b. Notice that the area of the circle inscribed in the square is close to the area of the octagon, perhaps slightly larger. Hence we might approximate the area of the circle by 64 square units. How close is this to the exact area of the circle?

Problems (cont'd)

3. Continuation:
   c. The previous observation led to the conclusion that the area of a circle of diameter d is well approximated by the area of a square with each side of length $\frac{8}{9}d$. Explain this conclusion.
   d. Show that saying that the area of a circle is equal to the area of a square with sides of length eight-ninths the diameter of the circle is equivalent to taking π equal to

$$\left(\frac{16}{9}\right)^2 \approx 3.1605.$$